# GEOMETRIC Constructions and Investigations with a Mira®

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# Introduction

In the opinion of the authors, one of the most significant documents written in the last twenty years is the National Council of Teachers of Mathematics (NCTM) publication *Curriculum and Evaluation Standards for School Mathematics.* Here is a quote from page 7 of this publication:

> First "knowing" mathematics is "doing" mathematics. A person gathers, discovers, or creates knowledge in the course of some activity having a purpose. The active process is different from mastering concepts and procedures. We do not assert that informational knowledge has no value, only that its value lies in the extent to which it is useful in the course of some purposeful activity. It is clear that fundamental concepts and procedures from some branches of mathematics should be known by all students; established concepts and procedures can be relied on as fixed variables in a setting in which other variables may be unknown. But instruction should persistently emphasize "doing" rather than "knowing that."

The Mira-oriented lessons contained in this book are certainly consistent with that point of view. These lessons require active participation on the part of students. The authors have used the related activities with students from a wide range of ages and these students have almost universally enjoyed using the Mira. It is a device that has high motivational value.

Geometry is receiving increased emphasis in the professional literature. However, many teachers are still asking the question "What should we teach and how should we teach it?" The following quote from page 112 of the *Standards* provide teachers with very good advice.

> Students discover relationships and develop spatial sense by constructing, drawing, measuring, visualizing, comparing, transforming, and classifying geometric figures. Discussing ideas, conjecturing, and testing hypotheses precede the development of more formal summary statements. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop formal arguments. The informal exploration of geometry can be exciting and mathematically productive for middle school students. At this level, geometry should focus on investigating and using geometric ideas and relationships rather than memorizing definitions and formulas.

While this advice is addressed primarily to middle school mathematics teachers, it is also practical for high school geometry teachers. This is particularly significant in the case of teachers of high school geometry courses in school systems where geometry is not emphasized in middle school. In such cases, some high schools

presently have courses entitled "Informal Geometry." The lessons in this booklet provide opportunities for students to construct, draw, visualize, compare, and transform in an informal setting.

## *What Is a Mira and What Can You Do With It?*

The main portion of the Mira is a piece of translucent red acrylic plastic about 9 cm by 15 cm. One of the 15 cm edges is rebated (beveled). The Mira is held upright by two ends also made of plastic. The purpose of the ends is to make the Mira sit perpendicular to the surface being examined. When the Mira is used, the rebated edge must be down and toward the user, and when any line is drawn along an edge of the Mira, it should be drawn along the rebated edge. Since the plastic is translucent, it is possible to see an object on the far side of the Mira in addition to seeing the image of a figure that is on the near side of the Mira.

The most obvious use of the Mira is for line reflections. The rebated edge of the Mira is put on the reflection line and the observed image can be sketched by tracing it on the far side of the Mira while viewing from the near side. Since it is possible to see through the Mira, it is easy to find symmetry lines of plane figures. Also, since the perpendicular bisector of a segment and the bisector of an angle are both symmetry lines, these constructions are trivial. A line perpendicular to a given line through a given point can be readily constructed. Thus, it is possible to use the Mira to construct the bisectors of the angles of a triangle, the altitudes of a triangle, and the perpendicular bisectors of the sides of a triangle. The Mira can be used to construct a line parallel to a given line. Congruent coplanar circles can be identified by making the image of one circle match the other circle. In the case of other coplanar figures such as segments, triangles, and quadri- laterals, congruence can be determined with a Mira only when one figure is the reflection image of the other.

## *Using the Mira*

It is very easy to learn to use the Mira. Generally speaking, the reflection of some figure or part of a figure is required. When this is the case, the Mira is placed between the figure being reflected and the general desired position of the image. The paper is then turned so that the figure is on the side of the Mira closest to the user. Two important points were made above but should be emphasized.

1. **When using the Mira, always place the beveled edge down and toward you.**
2. **When drawing a line along the edge of the Mira, always draw it along the beveled edge.**

## *Using this Book*

This book contains twenty-one geometry lessons in which extensive use of a Mira is required. The general format of each lesson involves some information for the teacher concerning the lesson and some student worksheet pages. The information for

the teacher includes a list of prerequisite lessons, a description of needed materials, and some suggestions to the teacher about conducting the lesson. Needed materials usually include only worksheets for each student and a Mira for each student, but occasionally a ruler/straightedge for each student is required, and a few lessons involve the use of a transparency. When a transparency is used, the master of the transparency is included with the materials for the lesson. The edge of the Mira can be used as a straightedge, but it is a little clumsy to use the Mira in this way, so it is recommended that a ruler be used to draw lines.

For each lesson, the transparency and the worksheet pages are numbered to correspond to the number of the lesson. For example, the transparency for Lesson 1 is labeled "Transparency 1," while all the worksheet pages are labeled "Worksheet 1." Worksheet pages after the first one are noted by the word "continued." Since Lesson 2 does not require a transparency, the book does not contain a transparency labeled "Transparency 2."

The "Directions for the Teacher" comments refer to lessons that are to be taught in a large group format with each student having a Mira. Minor adjustments can be made so that the materials can be used with individual students or with small groups of students. In this case, each student or small group of students should be given copies of all transparency pages in addition to all worksheet pages. Depending upon the particular lesson, some verbal directions may be necessary. It is desirable for each student to have a Mira in all learning situations. However, when this is not possible, a pair of students may share a single Mira. If students share a Mira, the teacher should check to see that the Mira is actually shared and both students complete the activity. The "Prerequisite Lessons" suggestions give general directions for sequencing the lessons. The lessons do not necessarily need to be taught in the order in which they appear.

The lessons have varying degrees of difficulty. The first 14 lessons are fairly easy, but lessons 15–21 are quite difficult.

# Lesson 1

# Introduction to the Mira

## *Materials Needed*

One copy of each of the following pages for each student: pages 3, 4, 5, and 6

One transparency of page 2

One Mira for each student

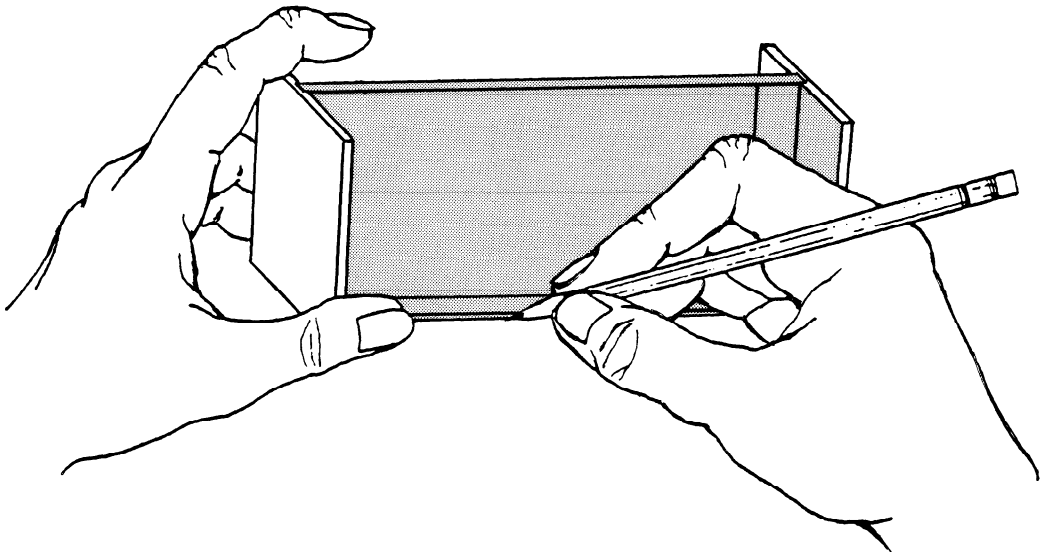
## *Directions for the Teacher*

Place the transparency on the overhead projector and go through the information given there. Be certain that your students can identify the rebated edge. Emphasize the importance of keeping the rebated edge down and toward them. Also emphasize that when they are to draw a line along an edge of the Mira, it must always be drawn along this rebated edge. This procedure is not particularly important for Lessons 1 and 2, but it is vital for many of the other lessons.

Distribute the first page of Worksheet 1. To complete this activity, the student should place the Mira between the child and the swing, turn the worksheet so the child is on the front side of the Mira, adjust the Mira so the image of the child is on the swing, and then trace the child on the swing. Distribute the other three pages of Worksheet 1. Do not staple these three pages together but give them out individually. Have the students work on one page at a time; when students have finished with one page, direct them to put that page aside. Provide individual assistance as needed.

# Transparency 1

The main portion of the Mira has a rebated edge. When using the Mira, place the rebated edge down and toward you so that lines are drawn as shown below.



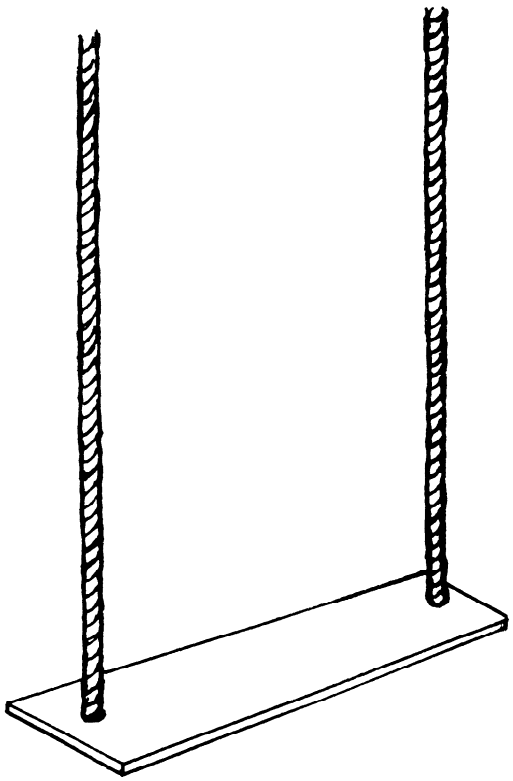
To avoid irregularities, it is recommended that a single sheet be used at a time.

Name _____ Date _____

Worksheet 1

# Introduction to the Mira

1. Use the Mira to put the child on the swing. Then reach behind the Mira and trace the child on the swing.

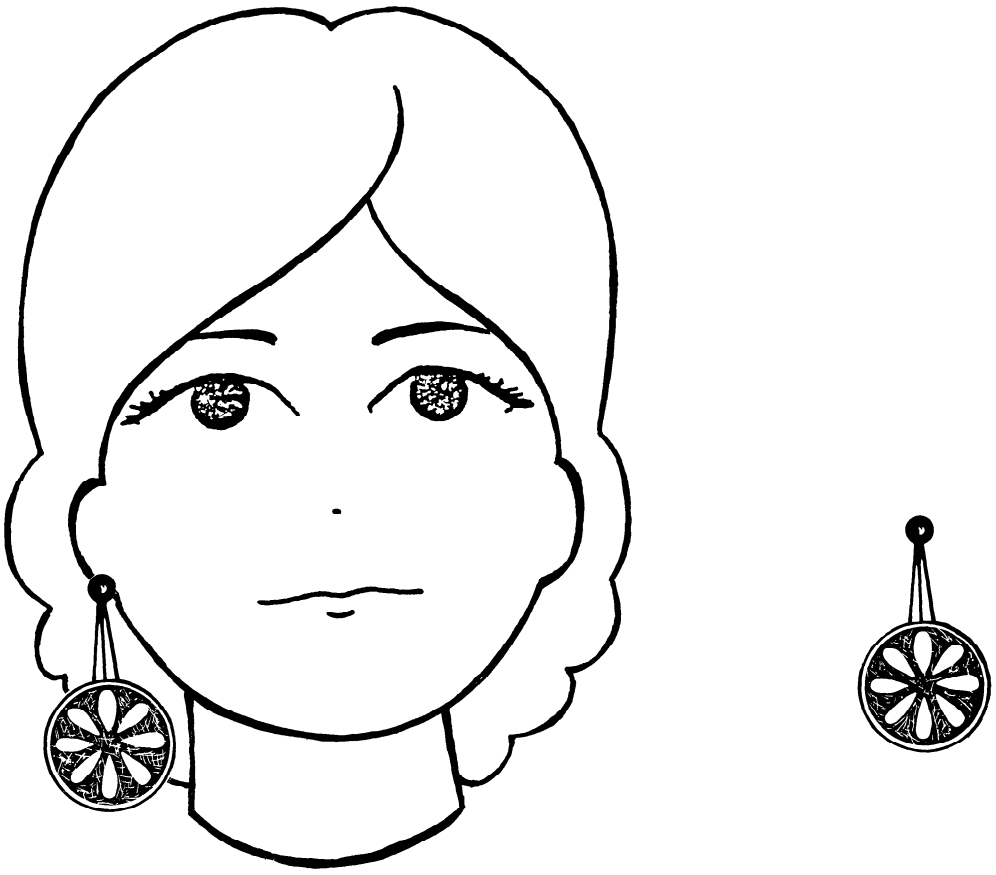


Name _____ Date _____

Worksheet 1

# Introduction to the Mira *(continued)*

2. Using the Mira, put the earring on the woman. Then reach behind the Mira and trace the earring on the woman.

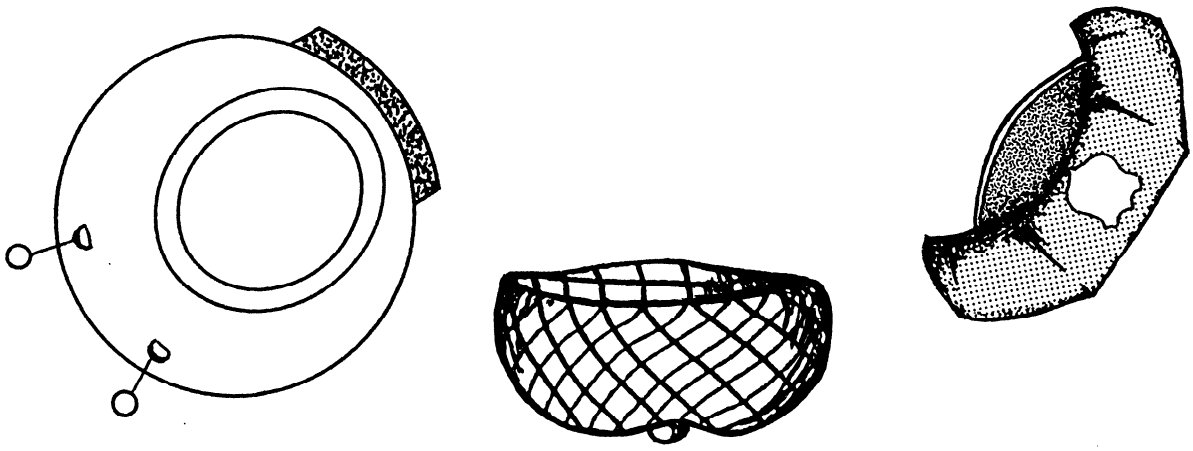


Name _____ Date _____

Worksheet 1

# Introduction to the Mira *(continued)*

3. Using the Mira, put each hat on the woman. Select the hat you like best and trace it on the woman.

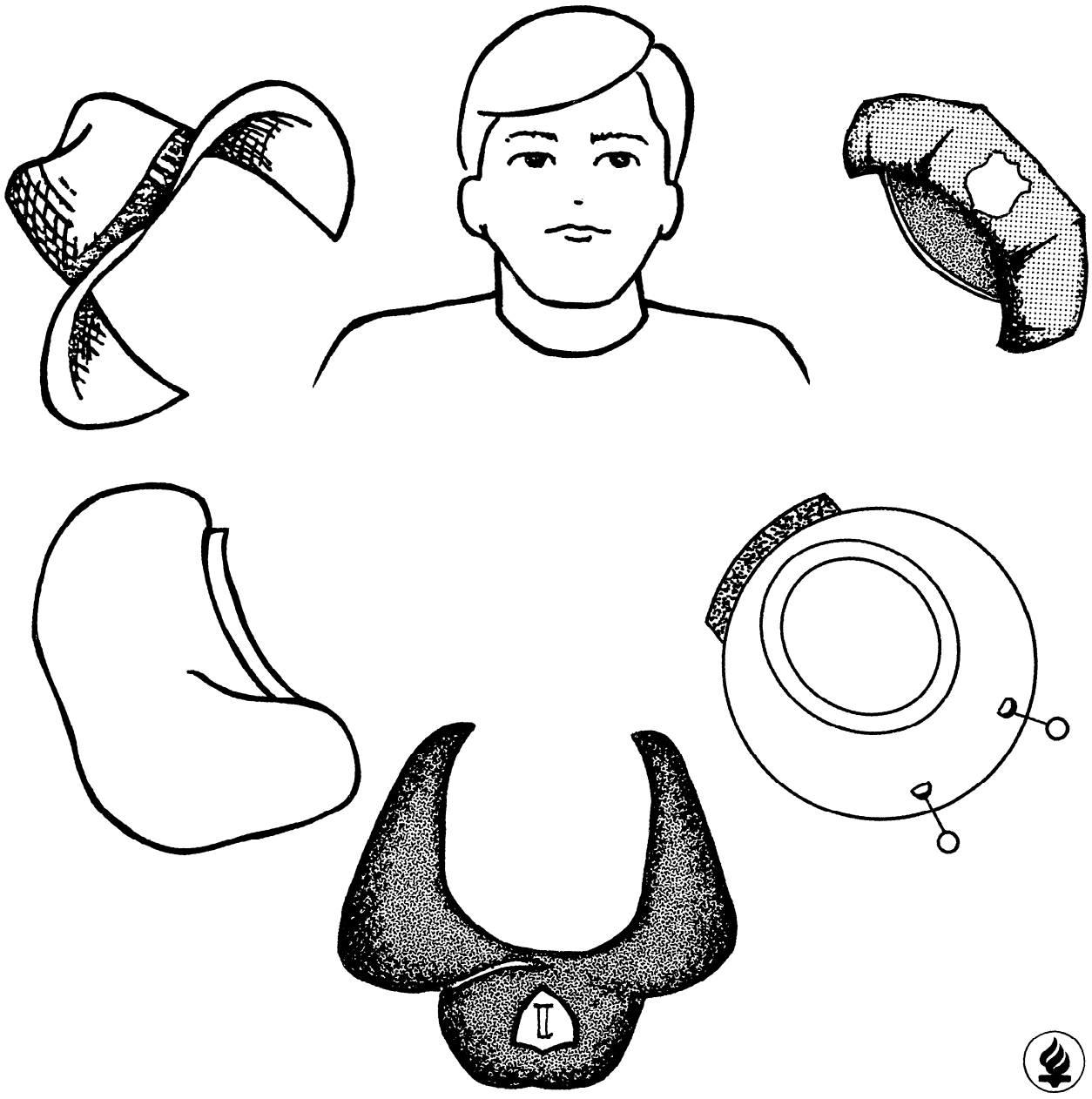


Name _____ Date _____

Worksheet 1

# Introduction to the Mira *(continued)*

4. Using the Mira, put each hat (helmet, cap) on the man. Select the hat you like best and trace it on the man.



Lesson 13

# Circles and Arcs

## *Prerequisite Lessons*

Lessons 1 and 4

## *Materials Needed*

One copy of page 54 for each student

One Mira for each student

## *Directions for the Teacher*

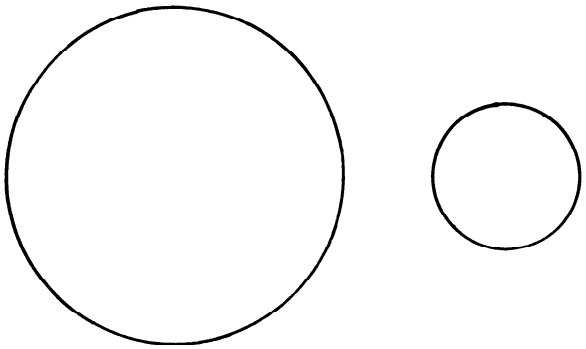
Distribute the worksheet and have the students start working on the problems presented there. They probably will be able to complete Problems 1, 2, and 3 without assistance. When they have finished Problem 1, you might want to point out that the line through the centers of the circles is actually the symmetry line. As they get to Problem 4, you will probably need to tell them that to quadrasect the arc, they will need to separate the arc into four congruent arcs. This can be done by bisecting the arc and then bisecting the two smaller arcs.

Name _____ Date _____

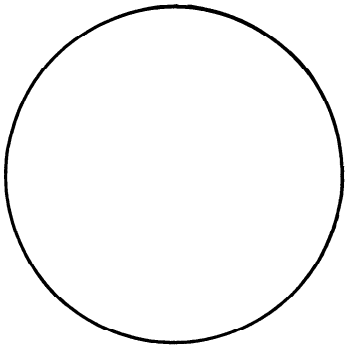
Worksheet 13

# Circles and Arcs

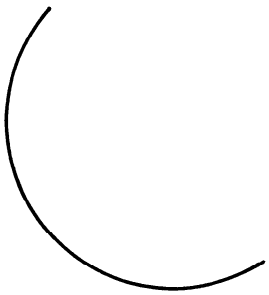
1. Use the Mira to construct the line that passes through the centers of both circles.



2. Use the Mira to find the center of the circle.



3. Use the Mira to bisect the arc.



4. Use the Mira to quadrasect the arc.