

***Learning Activities***  
***from the***  
***History of Mathematics***

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**J. Weston Walch, Publisher**

Portland, Maine

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# Introduction

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In suggesting new mathematics curricula and instruction methodologies to accommodate America's transition from an industrial to an informational society, the National Council of Teachers of Mathematics (NCTM) emphasizes the importance of historical perspectives in mathematics teaching. Among the new goals for students listed in the Council's *Curriculum and Evaluation Standards for School Mathematics*, the very first goal, "Learning to Value Mathematics," specifies:

*Students should have numerous and varied experiences related to the cultural, historical, and scientific evolution of mathematics so that they can appreciate the role of mathematics in the development of our contemporary society and explore relationships among mathematics and the disciplines it serves: the physical and life sciences, the social sciences, and the humanities. . . . It is the intent of this goal—learning to value mathematics—to focus attention on the need for student awareness of the interaction between mathematics and the historical situations from which it has developed and the impact that interaction has on our culture and our lives.*

Quite simply these statements affirm the importance of associating the teaching of mathematics with its history.

This concept is not new; it has been around for a long time. A century ago (1890) in marking another societal transitional period—that of moving from an agricultural society to an industrial one, and the place of mathematics in that movement—J. W. L. Glaisher, president of the British Association for the Advancement of Science, noted that "no subject loses more than mathematics by any attempt to dissociate it from its history." Acknowledging this concept and putting it into action are two different things, however. While influential people and organizations can proclaim the importance of the history of mathematics, it remains for us as classroom teachers to implement this association. Many teachers have indeed taken up the task. The NCTM's first Yearbook, published in 1926, describes the efforts of an Ohio teacher who challenged her 7th- and 8th-grade students with such questions as "Who was Thales?" and "Where did the signs  $+$ ,  $-$ ,  $\times$ ,  $\div$ , and  $=$  come from?" Such questions would be equally challenging and interesting today. However, efforts to use a historical outlook are often isolated and the result of the creative outlook of individual teachers. Neither methods courses nor texts usually discuss the techniques or pedagogy of incorporating historical material with the teaching of mathematics.

For many years I have worked with teachers and students in trying to overcome this problem. Together we have learned some of the methods and strategies that successfully incorporate mathematics with its history—strategies and methods worth sharing. This manual is intended for the purposes of that sharing. It is a reference handbook with suggested

activities and materials to use, problems, and bibliographies that should be of value to any mathematics teacher who wishes to enrich his or her teaching. As such, this book is not comprehensive but is rather intended to be a resource guide of ideas that you the teacher can use or modify depending on your particular needs. The contents discuss what has worked in many classrooms. We hope they will work in many more.

—*Frank Swetz*

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## CHAPTER 2

# The People of Mathematics

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While many terms have been applied to mathematics—an art, a science, the Queen of the Sciences, a game played with rules we do not fully understand—ultimately it is a human experience. Far too often the humanity of mathematics is forgotten in its classroom teaching. Students may learn some mathematical facts and some rules and procedures by which to apply them to a set of numbers or a geometrical figure, but frequently students are not quite sure just why and for what purpose these rules and procedures have been derived. This is where the history of mathematics can most effectively come into play: It can associate names, faces, and situations with particular concepts and procedures; it can identify the results learned in class as a human effort directed at solving a human need and associated with a particular person or group of people. Mathematics should not be anonymous. It should have an identity associated with people.

True, students may recognize some historical names such as Euclid, Pythagoras, or Descartes, but unfortunately, students' further understanding of the significance of these names is usually limited. To know that Euclid (ca. 300 B.C.) has something to do with geometry (no, he did not invent it) is reassuring, but how much more meaningful it would be if those students realized that Euclid's 13 books on *Elements* cover far more than geometry and represent the consolidation and systematization of 300 years of Greek mathematical experimentation and discovery. Aha! Mathematics occasionally needs to be consolidated and organized—a major realization. Associating Pythagoras with the right triangle is noteworthy, but better yet is the recognition of the Pythagoreans' concern with numerical harmony and their eventual confrontation with irrational numbers. René Descartes (1596–1650) may be credited with the popularization of the rectangular coordinate system, but how much more satisfying it would be to learn how he employed a concept of coordinates to bind geometry together with algebra and develop the discipline of analytic geometry. You can and should unfold these scenarios before your students.

The drama, suspense, and intrigue of mathematics—features that appeal to most audiences, including students—most often rest with the people of mathematics, the doers and movers in the history of mathematics. The Tartaglia–Cardano dispute demonstrates human foibles but also highlights the importance of algebraic computation in the 15th century. The failed attempts of Hippocrates of Chios (ca. 440 B.C.) to square the circle and solve a classical problem of Greek antiquity illustrate ingenuity and mathematical insights that were of value

in themselves. Hippocrates' quest was carried on by other mathematical adventurers including Leonardo da Vinci (1452–1519). Was this problem ever solved? If not, why? Did François Viète's (1540–1603) concern with secret codes figure in his attempts to develop a symbolic language for algebra? Why were the letter-writing efforts of Marin Mersenne (1588–1648) a stimulus to mathematical advances in the 17th century? Who "invented" the calculus first, Isaac Newton (1642–1727) or Gottfried Wilhelm Leibniz (1646–1716), and why did this controversy split the European mathematical community? Who was the mysterious Monsieur Le Blanc, and what was his relationship to Carl Friedrich Gauss (1777–1855)? What is the Russell paradox, and who is the Gödel of Gödel's proof? In seeking answers to such questions, a student learns much about mathematics and develops a fuller appreciation of its significance. Techniques you can use with a class that will help to acquaint students with the people of mathematics include using anecdotes and brief historical sketches and assigning reports and research projects.

### *Using Anecdotes and Brief Historical Sketches*

The injection of one or more brief historical anecdotes during a class discussion offers a change of pace from the usual teacher–student interaction. It can be amusing, informative, and thought-provoking and set the stage for further mathematical discussion and learning. As an example, consider the story involving the young Gauss, the so-called Prince of Mathematicians, who at age ten was given the task by a teacher of finding the sum for the natural numbers from 1 to 100. His teacher thought this problem would keep the class occupied for some time (busywork is not new) and was amazed when Gauss quickly made a mental calculation and supplied the correct answer, 5,050. The young boy had noted this pattern:

$$\begin{array}{r} 1 + 2 + 3 + \dots + 98 + 99 + 100 \\ + (100 + 99 + 98 + \dots + 3 + 2 + 1) \\ \hline 101 + 101 + 101 + \dots + 101 + 101 + 101 = 100(101) \end{array}$$

Realizing he had obtained twice the required sum, Gauss modified his answer by dividing by 2:

$$\frac{(100)(101)}{2} = 5,050$$

This simple story emphasizes the importance of looking at mathematics: seeking patterns and, if possible, using those patterns to solve a problem. While a powerful lesson in problem-solving has been carried out, further considerations and questions easily evolve: "Does the technique work for any consecutive series of numbers?" "Would it work for a series of even numbers?" "Can the class generalize a formula to encompass the process?" and so on.

Numerous other anecdotes are available to reflect on varied aspects of mathematics. Consider the instance when Johannes Müller (1436–1476), an accomplished German mathematician sometimes known as Regiomontanus, was challenged to a contest by a draftsman who wished to test Müller's geometric ability. The contest was the ruler and compass

construction of right triangles—the man who correctly constructed the most right triangles within a fixed period of time would be the winner. Müller easily won by initially constructing a semicircle and rapidly inscribing triangles in it using its diameter as a fixed side of the required triangles. He utilized the principle that all triangles inscribed in a semicircle are right triangles. While this story is not profound in itself, it reinforces and amplifies the right triangle/semicircle concept.

The importance of inquisitiveness combined with observation and mathematical experimentation can be conveyed by describing the 1581 experiences of Galileo Galilei (1564–1642) in observing a swinging altar lamp in the cathedral at Pisa. He timed the period for an arc of the lamp's swing by using his own heartbeats as a measure of time and discovered that the period of oscillation for a swinging body was independent of the arc length traveled. This principle became the basis for the development of the pendulum clock. Galileo employed such experimental practices throughout his scientific life. His dropping of two different-sized weights from the leaning tower of Pisa to resolve the law that the distance a body falls is proportional to the square of the time in flight, irrespective of the weight, has inspired several of my students to duplicate this experiment. (Much to their satisfaction and my relief, they affirmed Galileo's findings.)

Every teacher can easily accumulate a few favorite anecdotes relevant to his or her teaching situation, ones that pertain to the mathematics being taught and reveal the human side of mathematics while telling the listener something about mathematics itself.

Anecdotes are usually told orally, which demands a teacher's time and preparation. An easier method of bringing the people of mathematics to a class is through the use of brief historical sketches. The historical sketch is a printed presentation that identifies a mathematician or a mathematical accomplishment and then goes on to tell the reader some facts about the subject. Such sketches, when dealing with people, are often accompanied by a picture of the particular person in question. A sketch must be brief (one printed page), concise, easy to read, and appealing in some manner. Cartoons are frequently incorporated. Sketches like these lend themselves to use as classroom posters. Sets of sketches can be commercially purchased or constructed by teachers and students. In fact, a worthwhile learning project is to have students construct a set of historical posters that will eventually be displayed for viewing.

A sample set of historical poster sketches are included in the following pages. They are intended for possible classroom use. This set was devised and written with several learning factors in mind:

1. Each person depicted and discussed made an important contribution to the development of mathematics.
2. The contributions described span the various branches of mathematics and are chronologically distributed over the history of mathematics.
3. The selection of personages is representative of the diverse, multicultural nature of mathematical contributions.

The posters can be a source of either passive learning (students can read them individually as they wish) or active learning (direct questions can be asked and activities focused around a poster's subject). For illustrative purposes and possible use, a brief set of sample questions is included for each poster.

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## Thales (ca. 640–546 B.C.)

### *The First Mathematician*

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*About 640–546 B.C.*

Photo courtesy of the New York Public Library

Of course it is impossible really to name the *first* mathematician, that is, the very first person who used mathematics extensively in daily life. Perhaps this person was a tribal leader who kept a careful count of the wild game his or her hunting band had bagged or a shaman who carefully noted the positions of the moon as an indicator for the people to migrate to better pasturelands. Whoever this person was, he or she remains unknown. Written history does, however, recognize a single individual as the first mathematician. He is the Greek named Thales of Miletus who lived around 600 B.C. At that time, Miletus was a flourishing Greek colony on the coast of Asia Minor and a thriving trading center. Perhaps Thales' family were merchants, but he himself appears to have been a self-made man who used his talents to great advantage in the worlds of both commerce and science and mathematics.

Many stories are told about Thales. One story reveals that as a young man Thales was very poor and a friend, sharing his poverty, lamented on their status by remarking, "If one is born poor, then one will remain poor for the rest of his life." Thales disagreed strongly, saying that if he set his mind to the task, he could become rich. He challenged his friend to give him six months to accomplish this feat. Six months later, the friend returned to find Thales a wealthy man. He had foreseen a good olive harvest and went about the region leasing all available olive presses, the devices needed to make olive oil, an important food in the Greek diet. When the



good crop came in, Thales controlled all the olive presses and in turn the production of olive oil. The fees he charged for rendering the olives into oil with his presses made him a wealthy man and established him as both an entrepreneur and a merchant.

As a merchant, Thales traveled widely and became a keen observer of the customs and knowledge of neighboring peoples. In Babylonia he learned the techniques of astronomy, and in Egypt he was impressed by the geometric skills of the priest-surveyors who reestablished land boundaries after each annual flooding of the Nile River. While in Egypt, Thales demonstrated his own geometric skills by determining the height of the Great Pyramid of Giza by methods of indirect measurement. By comparing the pyramid's shadow with that of an object of known height and utilizing the property of similar triangles that corresponding sides are in proportion, Thales was able to find the unknown height.

Eventually, he returned to Miletus, retired from the commercial world, and devoted himself to studying and teaching philosophy, mathematics, and science. He brought together all the information he had learned about foreign mathematics and developed it into a systematic body of knowledge.

In particular, history credits Thales with providing proofs for several basic geometric propositions:

1. A circle is bisected by its diameter.
2. Every angle inscribed in a semicircle is a right angle.
3. If two straight lines intersect, the opposite angles formed are congruent.
4. The base angles of an isosceles triangle are equal.
5. The sides of similar triangles are proportional.
6. Two triangles are congruent if they have two angles and a side respectively congruent.

It was through such efforts that Thales became known as the first mathematician. Further, Thales was greatly admired for his wisdom and is supposed to have originated the saying that in seeking wisdom one must first “know thyself.” Once when asked what was the strangest thing he had seen in all his travels, he replied, “An aged tyrant.” Thales never married. He was further honored by later Greek generations by being chosen as the first of the Seven Wise Men of Antiquity, the only mathematician in this famous company.



## Questions and Investigations

### Thales of Miletus

The following questions and investigations are intended to broaden a historical understanding of the persons and issues examined. They should provoke thinking and guide further research on the subjects under discussion. In several instances, library resources may have to be consulted.

1. Thales was born in Miletus in Asia Minor. Today, in what country would we find the birthplace of Thales?

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2. In Egypt, Thales observed priest-surveyors at work. Why would priests be doing surveying?

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3. How do you think Thales used geometric knowledge to determine the height of the Great Pyramid? Remember, he could not climb to the top of the pyramid.

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4. Thales said that in seeking wisdom you must first “know thyself.” What did he mean?

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5. Thales was one of the Seven Wise Men of Antiquity. Name three other men in this group.

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## Pythagoras (ca. 580–500 B.C.) and the Pythagoreans

### *Mathematics and Mysticism*

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*About 580–500 B.C.*

Pythagoras was born about 580 B.C. on the island of Samos in the Aegean Sea. He left home at an early age and wandered for many years throughout the countries of the East, perhaps traveling as far as India. When he was approximately 50 years old, he returned home, settled in Crotona, a Greek colony in southern Italy, and founded a school devoted to the study of a mystical philosophy based on the concept that everything is controlled by numbers—whole numbers.

Pythagoras' teachings attracted student disciples who formed into a brotherhood or fraternity. The brotherhood had a strict set of rules and rituals. Pythagoreans, as they were called, could not eat meat or beans, drink wine, wear wool, or touch a white rooster. They believed in the transmigration of souls, thus the rules against eating meat and wearing wool—the animals these products came from may have been former friends. One story relates how Pythagoras came upon a man beating his dog and urged him to stop, saying, "In this dog lives the soul of my friend; I recognize him by his voice." The man stopped beating his dog.

In seeking mathematical harmony with the universe, the Pythagoreans studied *arithmetic*, the properties of numbers (number theory), music, astronomy, and geometry. Each of these subjects depended on the use of numbers. For example, Pythagoras taught that all matter was composed of four elements: fire, represented by the number 1; water, 2; air, 3; and earth, 4. These were the *holy tetrads*.

Photo courtesy of the New York Public Library



whose sum ( $1 + 2 + 3 + 4 = 10$ ) was the number of the universe. Numbers could have many meanings, which in turn were used to form relationships. For example, 2 was the female number; joining it with 3, the male number, resulted in 5, the number for marriage; 6 was the number for creation (the universe was created in six days); 4 represented justice (a square deal). Also, 6 was a perfect number, being the sum of its proper factors ( $1 + 2 + 3 = 6$ ); numbers smaller than the sum of their proper factors were called deficient numbers, and those larger were called abundant numbers. Pairs of numbers for which each is the sum of the proper factors of the other number were called amicable numbers. For example, 220 and 284 are amicable numbers. The Pythagoreans were also involved with figurative numbers, that is, numbers formed by a geometric arrangement of dots. The geometric arrangement lent itself to devising names for the numbers. Thus, there were triangular numbers (1, 3, 6, . . .), square numbers (1, 4, 9, . . .), pentagonal numbers (1, 5, 12, . . .), and so on. The sign for the society of Pythagoreans was the five-

pointed star, or pentagram. This geometrical figure had mystical properties: A series of smaller pentagrams could be constructed inside a given pentagram, and the legs of a pentagram are divided numerically according to the golden ratio.

In the history of mathematics, the Pythagoreans are noted for two major discoveries: the existence of irrational numbers, that is, numbers whose magnitude could not be expressed as a ratio of two whole numbers, and the numerical relationship for the sides of a right triangle. This latter accomplishment is usually called the Pythagorean theorem, but it is now known that Pythagoras and the Pythagoreans were not the first to discover this geometric theorem.

Legend relates how Pythagoras was killed by a mob when he was about 80 years old—he could have escaped, but refused to tread on a sacred bean field. The school he founded and the Pythagorean brotherhood continued for several centuries after the master's death.



## Questions and Investigations

# Pythagoras and the Pythagoreans

The following questions and investigations are intended to broaden a historical understanding of the persons and issues examined. They should provoke thinking and guide further research on the subjects under discussion. In several instances, library resources may have to be consulted.

1. The Pythagoreans believed in the transmigration of the soul. What does this mean? Do any people today also believe in this principle?

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2. Pythagoras thought that music was ruled by numbers. Do you agree with him? Explain your answer.

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3. Find two perfect numbers other than 6. List three deficient and three abundant numbers.

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4. If the Pythagoreans were really not the first people to know that the sum of the squares of the sides of a right triangle equal the square of the hypotenuse, who probably were the first people to know this important mathematical fact?

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5. The discovery of irrational numbers by the Pythagoreans caused them great concern. Why should this fact have troubled them?

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