

33 STEPS TO ALGEBRA READINESS

Fred Pirczak
illustrated by **Scott W. Earle**

 **J. WESTON
WALCH**
PUBLISHER
PORTLAND, MAINE

Contents

| | |
|--|-------------|
| <i>How to Use 33 Steps to Algebra Readiness</i> | <i>vii</i> |
| <i>Answer Keys</i> | <i>ix</i> |
| <i>Answer Sheet for Diagnostic Tests</i> | <i>xxi</i> |
| <i>Answer Sheet for Multiple-Choice Survey Test</i> | <i>xxii</i> |
| Diagnostic Test 1: Order of Operations in Expressions | 1 |
| Practice Sheet 1: Riddle | 2 |
| Diagnostic Test 2: Translating English Statements into Symbols | 3 |
| Practice Sheet 2: Scrambled Words of Wisdom | 4 |
| Diagnostic Test 3: Translating Sentences About Inequalities | 5 |
| Practice Sheet 3: Soccer Field Maze | 6 |
| Diagnostic Test 4: Solving Equations with One Unknown— Multiple-Choice | 7 |
| Practice Sheet 4: Crossnumber | 8 |
| Diagnostic Test 5: Solving Equations with One Unknown— Open-Ended | 9 |
| Practice Sheet 5: Circle Maze | 10 |
| Diagnostic Test 6: Solving Inequalities with One Operation | 11 |
| Practice Sheet 6: Cowboy Riddle | 12 |
| Diagnostic Test 7: Solving Equations with Two Operations— Multiple-Choice | 13 |
| Practice Sheet 7: Punishment Riddle | 14 |
| Diagnostic Test 8: Solving Equations with Two Operations— Open-Ended | 15 |
| Practice Sheet 8: Downhill Race | 16 |
| Diagnostic Test 9: Recognizing Commutative Properties | 17 |
| Practice Sheet 9: Commutative Properties Bingo | 18 |
| Diagnostic Test 10: Recognizing Associative Properties | 19 |
| Practice Sheet 10: Talkative People Riddle | 20 |

| | |
|--|----|
| Diagnostic Test 11: Recognizing the Identity Properties and the Multiplication Property of Zero | 21 |
| Practice Sheet 11: Prospecting for Gold | 22 |
| Diagnostic Test 12: Understanding the Distributive Property | 23 |
| Practice Sheet 12: Hockey Puck Maze | 24 |
| Diagnostic Test 13: Identifying the Factors of a Number | 25 |
| Practice Sheet 13: Thinking About the Hereafter | 26 |
| Diagnostic Test 14: Identifying Prime Numbers and Composite Numbers | 27 |
| Practice Sheet 14: Viking Explorer | 28 |
| Diagnostic Test 15: Exponents and Powers | 29 |
| Practice Sheet 15: Pun Fun | 30 |
| Diagnostic Test 16: Identifying Greatest Common Factors and Least Common Multiples | 31 |
| Practice Sheet 16: Escalator Riddle | 32 |
| Diagnostic Test 17: Identifying Equivalent Fractions and Simplifying Fractions | 33 |
| Practice Sheet 17: Fraction Bingo | 34 |
| Diagnostic Test 18: Identifying Mixed Numbers and Equivalents | 35 |
| Practice Sheet 18: Storms in the Atlantic | 36 |
| Diagnostic Test 19: Multiplying Fractions and Mixed Numbers | 37 |
| Practice Sheet 19: Alphabet Check | 38 |
| Diagnostic Test 20: Dividing Fractions and Mixed Numbers | 39 |
| Practice Sheet 20: A Different Kind of Horse | 40 |
| Diagnostic Test 21: Adding and Subtracting Like Fractions and Mixed Numbers | 41 |
| Practice Sheet 21: A Friendly Thought About Seashells | 42 |
| Diagnostic Test 22: Adding and Subtracting Unlike Fractions and Mixed Numbers | 43 |
| Practice Sheet 22: Lost in the Woods | 44 |
| Diagnostic Test 23: Identifying Equivalent Fractions, Mixed Numbers, and Decimals | 45 |
| Practice Sheet 23: New Baby's Initials | 46 |

| | |
|--|----|
| Diagnostic Test 24: Adding and Subtracting Decimals | 47 |
| Practice Sheet 24: Find the Animal | 48 |
| Diagnostic Test 25: Multiplying Decimals | 49 |
| Practice Sheet 25: Words of Wisdom | 50 |
| Diagnostic Test 26: Dividing Decimals by Whole Numbers and Decimals | 51 |
| Practice Sheet 26: Wacky Definition | 52 |
| Diagnostic Test 27: Rounding Whole Numbers and Decimals | 53 |
| Practice Sheet 27: Emergency! | 54 |
| Diagnostic Test 28: Adding and Subtracting Negative Numbers | 55 |
| Practice Sheet 28: A Criminal Joke | 56 |
| Diagnostic Test 29: Multiplying and Dividing Negative Numbers | 57 |
| Practice Sheet 29: Flea Joke | 58 |
| Diagnostic Test 30: Solving Equations with Negative Numbers | 59 |
| Practice Sheet 30: Basketball Maze | 60 |
| Diagnostic Test 31: Translating Word Problems into Equations | 61 |
| Practice Sheet 31: Ark Riddle | 62 |
| Survey Test: Multiple-Choice | 63 |
| Survey Test: Open-Ended | 66 |
| Certificate of Achievement | 68 |

How to Use *33 Steps to Algebra Readiness*

Success in beginning algebra depends greatly on students' prior mastery of the skills that underlie success in algebra. Students who lack algebra readiness skills—such as the ability to work with negative numbers, identify the factors of a number, and translate word problems into equations—are likely to experience difficulty, if not failure, when they begin their study of algebra. Even students who have previously mastered the underlying skills may need brush-up reviews and practice. This book is designed to remedy these problems.

Ideally, the book should be used in math classes just prior to the introduction of algebra. It may also be used by algebra teachers during the first few weeks of algebra instruction with students who are underprepared. Although algebra teachers have much to cover, they will find that if they take the time at the beginning of the course to deal with crucial deficiencies, students will make quicker and more satisfactory progress during the rest of the semester.

The *Algebra Readiness* Diagnostic Tests in this book identify the specific algebra readiness skills students lack. Each test covers a specific skill or narrow cluster of skills that students should master prior to studying algebra. For each test, there is an *Algebra Readiness* Practice Sheet. Thus, when a deficiency is identified by a test, teachers can assign the associated practice sheet to correct the problem.

Characteristics of the *Algebra Readiness* Diagnostic Tests

Most of the tests contain 12 multiple-choice items. The incorrect choices are based on likely student mistakes. In addition, a fourth choice—NG (Not Given)—is provided. This choice provides an answer for students who make a mistake not anticipated by the test writer. Of course, the “Not Given” choice is sometimes correct; this shows students that they must consider this option seriously.

The multiple-choice format makes scoring easy, especially if you tell students to use the answer sheet provided on page *xxi* instead of marking their answers on the test sheets. By punching out holes for the correct answers in a copy of the answer sheet, a scoring overlay may be made. The overlay makes it easy to count the number of correct choices marked.

Note that it is not always necessary to begin with Diagnostic Test 1. Performance on the Survey Test described below and other sources of information may indicate the need to start at some other point for testing and instruction.

Interpretation of the Scores on the *Algebra Readiness Diagnostic Tests*

Even students with a high degree of proficiency in a skill covered by a test will occasionally make a mistake due to a clerical error, misunderstanding a question, or similar problem. Therefore, it is suggested that the acceptable mastery level for a skill be set at about 10 or 11 correct answers to the 12 problems. At this level, students have sufficient mastery of the skill for its use in algebra. Students who get only 8 or 9 items right need additional practice. Those who have only 7 or less right need both instruction and practice.

Characteristics and Use of the Survey Test

The last tests in this kit are two survey tests of all the skills covered in the Diagnostic Tests. The first is a multiple-choice test and the second is open-ended. Survey Test item number 1 is based on Diagnostic Test 1, Survey Test item number 2 is based on Diagnostic Test 2, and so on.

The Survey Tests give an overall indication of the need for instruction and practice in algebra readiness skills. In addition, they may be used as both a pretest and a posttest in a course designed to prepare students for algebra instruction, and may be used as a primary basis for assigning course grades.

Characteristics and Use of the *Algebra Readiness Practice Sheets*

A variety of formats are used on the practice sheets to maintain student interest—jokes, puns, mazes, wacky definitions, cross-numbers, connect-the-dots, and so on. Each sheet is self-checking; that is, only students who get the answers right can get the punch line, get through the maze, etc. This makes them ideal for homework assignments or independent in-class work.

The *Algebra Readiness Diagnostic Tests* aid you in selecting the practice sheets to assign. Assign only those that the tests indicate are needed. Before assigning a sheet, briefly instruct students in the skills that they will need to use. For example, before assigning Practice Sheet 29, review with students the rules for multiplying and dividing negative numbers. Because each sheet covers only a single skill or closely related cluster of skills, such instruction can usually be accomplished in a short amount of time.

The Certificate of Achievement

The last sheet in this book is a Certificate of Achievement. It may be duplicated and awarded to students for a variety of reasons, such as turning in all homework on time for a month, making the greatest gains from pretest to posttest, etc. It may also be used as an award for the two bingo contests in the package. Award the certificate to the first five players who find bingo.

DIAGNOSTIC TEST 1: Order of Operations in Expressions

Directions: Solve each expression and circle the letter of the correct choice. If the correct answer is not given, circle choice D.

1. $12 - (2)(5) + 3$
(A) 5 (B) 25 (C) 53 (D) NG
2. $10 + 5 + (3)(2)$
(A) 21 (B) 36 (C) 47 (D) NG
3. $9 - \frac{6}{2} + 6$
(A) $\frac{3}{8}$ (B) 3 (C) $7\frac{1}{2}$ (D) NG
4. $\frac{(6)(2)}{4} + (5)(2) - 5$
(A) -2 (B) 11 (C) 8 (D) NG
5. $2 + (5)(6) - (3)(5) + (6)(2)$
(A) 12 (B) 29 (C) 402 (D) NG
6. $[(15 - 4)(2)] [6 + 4]$
(A) 70 (B) 1,320 (C) 220 (D) NG
7. $(28)(15 + 9) - (36)(12)$
(A) -3 (B) 240 (C) 4,716 (D) NG
8. $(109 - 12)(3) + (22)(12 - 3)(0)$
(A) 291 (B) 489 (C) 2,425 (D) NG
9. $[366 + 10 - 2] [30 - (14)(2)]$
(A) 376 (B) 742 (C) 6,016 (D) NG
10. $\frac{[2 + (5)(6) - (9 - 3)]}{\left[\frac{(2)(8)}{(4+4)}\right]}$
(A) 13 (B) 4 (C) 10 (D) NG
11. $4 + (3)(2) + \frac{24}{2} - (2)(2)$
(A) 13 (B) 18 (C) 30 (D) NG
12. $6 + 4 + 7 + 8 + 12 \times \frac{4}{2}$
(A) 36.5 (B) 74 (C) 49 (D) NG

PRACTICE SHEET 1: Riddle

How can you keep a teenager out of hot water?

Answer: _ _ _ _ _ .

The answer has four words. Here's how to solve the riddle. Calculate an answer for each expression. Then find the answer in the Letter Box, and record the letter above that number. The letter above the first answer is the first letter of the first word. The second answer gives the second letter, and so on. Write the letters in the spaces above. *Note:* Only some of the letters in the Letter Box will be used.

1. $10 + (3)(3) + 4 = \underline{\quad}$

8. $[(9)(9 + 5)] [22 + 5] = \underline{\quad}$

2. $15 - \frac{4}{2}(3) = \underline{\quad}$

9. $(9 + 1)(6) \div 6 = \underline{\quad}$

3. $\frac{36}{6} + \frac{6}{6} - 2 = \underline{\quad}$

10. $\frac{(599 + 19 - 3)}{(3 + 2)} = \underline{\quad}$

4. $100 + (12)(2) + (5)(2) = \underline{\quad}$

11. $3 + (2)(87) + 3 = \underline{\quad}$

5. $2(6 + 120) - 5(26 - 7) = \underline{\quad}$

12. $[(5)(13 + 18)] [22 - 5] = \underline{\quad}$

6. $\frac{15}{(13-8)} + \frac{9}{(2+1)} = \underline{\quad}$

13. $[(3 + 5)(2 + 1)] [15 + 9] = \underline{\quad}$

7. $64 \div 8 + (15)(22 + 9) = \underline{\quad}$

Letter Box:

| | | | | | | | |
|-----|-----|-----|------|------|------|------|------|
| B | F | T | S | U | S | P | W |
| 0 | 4 | 5 | 6 | 9 | 10 | 23 | 43 |
| I | D | I | A | N | K | H | C |
| 123 | 134 | 157 | 165 | 180 | 336 | 438 | 458 |
| H | O | T | J | G | I | E | Y |
| 473 | 515 | 576 | 1411 | 2322 | 2635 | 3402 | 4693 |

DIAGNOSTIC TEST 2: Translating English Statements into Symbols

Directions: Circle the letter of the choice that translates each sentence into symbols. If the correct answer is not given, circle choice D.

1. Nine divided by three.
(A) $3 - 9$ (B) $\frac{3}{9}$ (C) $\frac{9}{3}$ (D) NG
2. A number X increased by twelve.
(A) $X + 12$ (B) $12X$ (C) $12 + 12X$ (D) NG
3. The sum of a number F and three minus four.
(A) $F + 3 + 4$ (B) $F + 7$ (C) $3F + 4$ (D) NG
4. From a number Y times two subtract two.
(A) $4Y$ (B) $2Y - 2$ (C) $Y - 2 - 2$ (D) NG
5. From ten subtract twelve and add a number N .
(A) $12 - 10N$ (B) $12 + 10 + N$ (C) $10 - 12 + N$ (D) NG
6. The product of six and a number Z divided into twenty-four.
(A) $\frac{6Z}{24}$ (B) $\frac{24}{(6Z)}$ (C) $\frac{X}{6+Z}$ (D) NG
7. Six added to the product of four and a number X .
(A) $6 + 4X$ (B) $6 + 4 + X$ (C) $6 + (4)(6) + 6X$ (D) NG
8. The quotient of a number N divided into twelve, plus three.
(A) $\frac{N}{12}$ (B) $\frac{12}{(N)(N)}$ (C) $\frac{12}{N} + 3$ (D) NG
9. Twelve more than a number Y divided by the product of three and a number X .
(A) $Y + 12 + 3X$ (B) $12 - \frac{Y}{3X}$ (C) $\frac{12Y}{3X}$ (D) NG
10. Six times the difference when ten is subtracted from a number F .
(A) $6(10 - F)$ (B) $6(F - 10)$ (C) $6F - 10$ (D) NG
11. Fifty minus the product of a number X and a number Y .
(A) $(50 - X)Y$ (B) $50 - XY$ (C) $50 - X + Y$ (D) NG
12. A number N subtracted from the quotient of a number X divided by a number Y .
(A) $N - \frac{X}{Y}$ (B) $\frac{N}{Y} - X$ (C) $\frac{X}{Y} - N$ (D) NG

PRACTICE SHEET 2: Scrambled Words of Wisdom

When unscrambled, some of the words in the box form a wise saying. Here is how to find the saying: First translate each statement into symbols and write them on the line. Then find the word above those symbols in the Word Box. The word that has the first answer under it is the first word in the saying, the second word has the second answer under it, and so on.

Write the unscrambled saying here: _____

- _____ 1. Twenty divided by ten.
- _____ 2. A number Y increased by twice Y .
- _____ 3. From three times a number Y subtract five.
- _____ 4. The product of a number Y and seven divided by ten.
- _____ 5. The quotient of a number Y divided into the product of five and four.
- _____ 6. Ten times the difference of nine subtracted from a number Y .
- _____ 7. Eight more than a number Y divided by two more than X .
- _____ 8. The sum of a number Y and twice a number X .
- _____ 9. From four times a number Y subtract four.

Word Box:

| | | | | | |
|-----------------------------|---------------------------------|----------------------|-------------------------------------|-----------------------|--------------------------|
| DON'T $\frac{10}{20}$ | TURNED $\frac{(8+Y)}{(X+2)}$ | PERCENT $Y+2+Y$ | SUCCESS $3Y-5$ | AND $5-3Y$ | |
| INSIDE $Y+2X$ | INSPIRATION $\frac{6+Y}{XN}$ | INTERRUPT $2Y+2X$ | WHEN $4-4Y$ | | |
| REMEMBER $\frac{20}{10}$ | THAT $Y+2Y$ | OUT $4Y-4$ | FROM $\frac{Y}{5+4}$ | IS $\frac{7Y}{10}$ | YOUR $Y+\frac{7}{10}$ |
| FAILURE $(10)(Y-9)$ | SIMPLY $\frac{(5)(4)}{Y}$ | ENVY $9(Y)-90$ | PERSPIRATION $\frac{X+2}{(8+Y)}$ | | |

DIAGNOSTIC TEST 3: Translating Sentences About Inequalities

Directions: Circle the letter of the choice that translates each sentence into symbols. If the correct answer is not given, circle choice D.

- Fifteen is greater than a number Y .
(A) $15 > Y$ (B) $15 < Y$ (C) $Y > 15$ (D) NG
- The product of three and a number X is less than twelve.
(A) $3X > 12$ (B) $3X < 12$ (C) $3 + X < 12$ (D) NG
- A number Y divided into a number X is less than one hundred.
(A) $\frac{X}{Y} > 100$ (B) $\frac{Y}{X} < 100$ (C) $X < Y < 100$ (D) NG
- Thirty more than a number N is greater than fifty.
(A) $50 > 30 + N$ (B) $50 + 30 > N$ (C) $30 + N > 50$ (D) NG
- Ten plus twice a number B is less than thirty.
(A) $10 + 2B > 30$ (B) $10 + 2B < 30$ (C) $(2)(10) + B < 30$ (D) NG
- A number G decreased by five is less than sixty.
(A) $G < 5 = 60$ (B) $G - 5 > 60$ (C) $G - 5 < 60$ (D) NG
- A number N is greater than the quotient of X divided by Y .
(A) $N > \frac{X}{Y}$ (B) $N > \frac{Y}{X}$ (C) $N < \frac{X}{Y}$ (D) NG
- Ten is greater than a number X and more than a number Y .
(A) $X < 10$ and $Y < 10$ (B) $X > 10 > Y$ (C) $X > 10$ and $Y > 10$ (D) NG
- The sum of seven and a number N is greater than the sum of ten and a number M .
(A) $7 + N < 10 + M$ (B) $7N > 10M$ (C) $7 + N > 10 + M$ (D) NG
- Eight subtracted from a number X increased by three is less than the product of six and a number Y .
(A) $8 - X + 3 > 6Y$ (B) $X - 8 + 3 < 6Y$ (C) $X - 8 - 3 < \frac{6}{Y}$ (D) NG
- A number W is less than sixty but more than fifty.
(A) $50 < W < 60$ (B) $60 < W$ and $50 < W$ (C) $60 < W < 50$ (D) NG
- The product of a number X and a number Y is greater than or equal to twenty-five.
(A) $X + Y \geq 25$ (B) $X \leq 25$ (C) $X \leq 25Y$ (D) NG

PRACTICE SHEET 3: Soccer Field Maze

Directions: First translate each sentence into symbols and write the translation on the line. Shade in all boxes that contain correct answers. Then draw a line from the ball to the goal side of the field going only through shaded boxes, which contain correct answers.

- _____ 1. Twenty is less than a number Y .
- _____ 2. The product of a number N and six is greater than twenty.
- _____ 3. Twenty more than a number Y is greater than fifty.
- _____ 4. Twenty plus twice a number N is less than fifty.
- _____ 5. A number Y decreased by twenty is greater than fifty.
- _____ 6. Six is greater than a number Y but less than a number N .
- _____ 7. A number N is less than thirty but more than twenty.
- _____ 8. Six subtracted from a number Y increased by twenty is less than or equal to the product of thirty and a number N .
- _____ 9. The product of six and a number Y is greater than or equal to thirty.
- _____ 10. The sum of a number N and six is less than or equal to the sum of a number Y and thirty.
- _____ 11. A number N divided into fifty is less than the sum of a number Y and two.

| | | | | | |
|----------|------------------------|---------------------|-----------------------|-----------------------|------------------------|
| G | $6 + N > 20$ | $Y - 20 > 50$ | $Y - 6 + 20 \leq 30N$ | $20 < Y$ | $6N > 20$ |
| O | $Y - 20 < 50$ | $6Y \geq 30$ | $6N < 20$ | $20Y < 50$ | $\frac{50}{N} < Y + 2$ |
| A | $\frac{N}{50} < Y + 2$ | $N + 6 \leq Y + 30$ | $20 - Y > 50$ | $20 + 2N > 50$ | $30 > N > 20$ |
| L | $Y + 20 > 50$ | $20 + 2N < 50$ | $6Y \leq 30$ | $Y - 6 + 20 \geq 30N$ | $N > 6 > Y$ |
| | $Y + 20 < 30$ | $N < 6 < Y$ | $20 > Y$ | $20 > N < 30$ | $Y + 20 < 50$ |



DIAGNOSTIC TEST 23:
Identifying Equivalent Fractions,
Mixed Numbers and Decimals

Directions: Circle the letter of the correct choice for each of the following. If the correct answer is not given, circle choice D. *Note:* Answers are expressed in lowest terms.

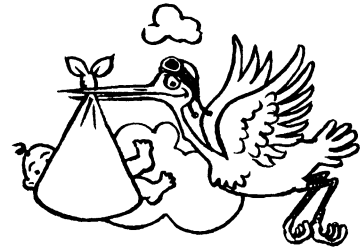
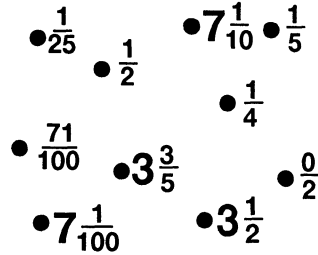
1. What does 0.5 equal?
(A) $\frac{1}{5}$ (B) $\frac{0}{5}$ (C) $\frac{1}{2}$ (D) NG
2. What does 0.6 equal?
(A) $\frac{1}{6}$ (B) $\frac{1}{2}$ (C) $\frac{3}{5}$ (D) NG
3. What does 0.25 equal?
(A) $\frac{25}{1000}$ (B) $\frac{1}{25}$ (C) $\frac{0}{25}$ (D) NG
4. What does 5.8 equal?
(A) $5\frac{4}{5}$ (B) $5\frac{5}{8}$ (C) $\frac{5}{8}$ (D) NG
5. What does 10.125 equal?
(A) $10\frac{1}{8}$ (B) $\frac{1}{5}$ (C) $10\frac{1}{25}$ (D) NG
6. What does 18.25 equal?
(A) $18\frac{2}{5}$ (B) $18\frac{1}{4}$ (C) $1\frac{8}{25}$ (D) NG
7. What does $\frac{4}{8}$ equal?
(A) 0.48 (B) 0.5 (C) 4.8 (D) NG
8. What does $\frac{9}{10}$ equal?
(A) 9.10 (B) 0.99 (C) 0.9 (D) NG
9. What does $3\frac{5}{50}$ equal?
(A) 3.1 (B) 3.55 (C) 35.50 (D) NG
10. What does $18\frac{9}{23}$ equal?
(A) 187.5 (B) 18.75 (C) 18.912 (D) NG
11. What does $14\frac{12}{24}$ equal?
(A) 1412.24 (B) 14.1224 (C) 14.5 (D) NG
12. What does $20\frac{75}{100}$ equal?
(A) 20.75 (B) 20.100 (C) 20.751 (D) NG

PRACTICE SHEET 23: New Baby's Initials

Here's how to find out what they are: First write your answers to the questions on the lines. Express answers in lowest terms. Then draw lines from the tip of the first stork's beak to the first answer, from the first answer to the second answer, and so on. When you can no longer find correct answers in the box for the first stork, start a line at the tip of the beak of the second stork. Three letters of the alphabet will emerge.

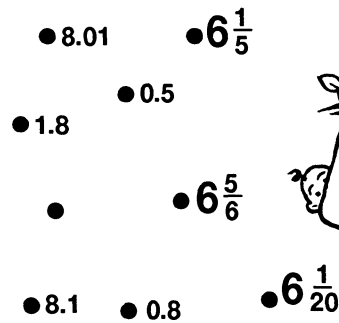
1. What is the fraction equivalent of 0.2? _____
2. What is the fraction equivalent of 0.25? _____
3. What is the fraction equivalent of 7.1? _____
4. What is the fraction equivalent of 3.6? _____

FIRST INITIAL:



5. What is the fraction equivalent of 6.05? _____
6. What is the decimal equivalent of $\frac{6}{12}$? _____
7. What is the decimal equivalent of $\frac{8}{10}$? _____

SECOND INITIAL:



8. What is the decimal equivalent of $\frac{90}{100}$? _____
9. What is the decimal equivalent of $7\frac{18}{36}$? _____
10. What is the decimal equivalent of $6\frac{3}{5}$? _____
11. What is the decimal equivalent of $6\frac{5}{25}$? _____

THIRD INITIAL:

