Middle School Math You Really Need

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Contents

Introduction .... *vii*

Chapter 1 **Math and Money**

Teacher Section and Answers 1

Reproducible Student Section 6–18

1. Simple and Compound Interest 6
2. Where Do You Use Percentage? How About Sales? 8
3. Using Math to Read the Newspaper's Financial Pages 10
4. "Sales!" 11
5. Buildings and Grounds 12
6. Taxes 13
7. Operating Costs of Electrical Appliance 16

Chapter 2 **Math for Travelers**

Teacher Section and Answers 19

Reproducible Student Section 24–40

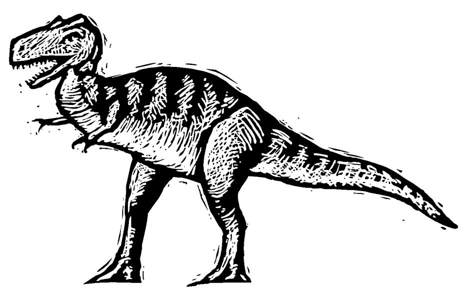
8. Change, Cambio, Wechsel: Changing Foreign Currency 24
9. Speed, Time, and Distance 26
10. Time Zones in the United States 29
11. Shortest Travel Distances Along the Earth 31
12. Weather Maps 34
13. Road Maps 36
14. Measuring Distance with a Wheel 38
15. MPG 40

Chapter 3 **Math and Sports**

Teacher Section and Answers 41
Reproducible Student Section 46–60
16. What's My Fielding Percentage? 46
17. What's My Earned Run Average? 49
18. In the Sports Pages 51
19. More in the Sports Pages 53
20. Still More from the Sports Pages 55
21. Math at the Olympics 57

Chapter 4 **Math with Big and Significant Numbers**

Teacher Section and Answers 61
Reproducible Student Section 67–79
22. Numbers That Mean Something: Significant Figures 67
23. Numbers for Astronomers 70
24. Models of the Solar System and the Universe 74
25. Big Bang, Life, Dinosaurs, Humans, and a Time Line 76
26. The Kuiper Belt 78



Chapter 5 **Math and Food**

Teacher Section and Answers 81
Reproducible Student Section 84–91
27. Nutritional Facts 84
28. Milk, Lowfat Milk, and Skim Milk 86
29. Math at the Grocery Store 89
30. Serving Food 91

Chapter 6 **Math in Many Places**

Teacher Section and Answers 93
Reproducible Student Section 100–117
31. Estimating and Approximating 100
32. Math and Property 102
33. Prime Numbers 106
34. Scaling 108
35. Velocity from Distance and Time 109
36. Are Negative Numbers Ever Real? 112
37. Handheld Calculators 114
38. Photographic Light Meters 116

Chapter 7 **Math in Art and Music**

Teacher Section and Answers 119
Reproducible Student Section 125–129
39. Math in Art 125
40. Mathematics in Music 127
41. The Mathematics of Rhythm 123

Introduction

In writing this book our goal was to provide a variety of real-world situations that would serve as a source of mathematical questions for middle-grade students. We wanted these youngsters to realize that mathematics really is used in solving problems that they are likely to encounter in their own lives.

At the same time, we were constantly guided by the standards recommended by the National Council of Teachers of Mathematics and by our desire to meet the needs of teachers who use cooperative learning in their classrooms. As a result, you will find that much of the material in this book fosters the ideas and suggestions found in *Curriculum and Evaluation Standards for School Mathematics* (1989), recommended by the working groups of the Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics. In particular, we have used "problem solving as a means as well as a goal of instruction," as recommended in the *Standards.* We have certainly tried to encourage the active involvement of students in making use of mathematics; we have suggested cooperative approaches to learning so that students can teach one another and learn from one another; we have encouraged students to communicate ideas through discussion and writing as well as by calculations; and we have assumed that students will make use of pocket calculators to avoid tedious calculations.

Cooperative Learning

The National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics* for middle grade students includes the statement, "Working in small groups provides students with opportunities to talk about ideas and listen to their peers, enables teachers to interact more closely with students, takes positive advantage of the social characteristics of the middle school student, and provides opportunities for students to exchange ideas and hence develops their ability to communicate and reason." A number of studies have shown that students who work together achieve more and are more capable of solving problems. They are also more helpful to, and caring of, one another and develop better self-esteem and social skills. These studies also reveal that tutoring benefits both tutor and tutee and increases time spent on a task, which is probably the most important factor in learning.

In most professions today, be it business, science, publishing, medicine, or whatever, the problems encountered are complex and arriving at their solutions requires cooperative efforts. Since students will enter a workplace where cooperation is essential, cooperative learning would seem an appropriate approach to use in schools.

Teachers and Cooperative Learning

This book was written with the assumption that the children using it would be working in cooperative groups. We have assumed that the groups will be made up of four students who are frequently broken into pairs. We chose four as a group size because it offers twice as many ways to form pairs (6) as does a group of three; it allows for a group consisting of one high achiever, two medium ability students, and one low achiever, and it avoids the "odd man out" situation. Such an arrangement gives you, the teacher, the potential for a teaching aide (the high-achiever) for each group.

Of course, as teacher, you know the children and their needs better than we do. Consequently, you may decide on a different group size or structure if you use cooperative learning. If you prefer to have students work individually rather than in groups, you can still make use of the problems provided in this book. All you need do is cover the initial instructions to students when you make copies of the pages that contain problems.

Choosing Groups

One way to establish groups for cooperative learning is for you to assign students so that each group has the diversity in ability mentioned above. As the one who best knows the students, you can probably establish groups that will work well together. In addition to ability, you probably will want to ensure that the groups are diverse with regard to gender, race, and ethnicity as well.

On the other hand, you may prefer to give students some role in the selection process. One way to do this is to take aside the top quarter of the class in terms of ability and tell them that they are going to be captains of new work groups. Give them the names of the students who are in the lowest achievement quarter and tell them they are each to choose one person from that group. Then the two members of each growing group can select members from among the remaining average ability students. You will probably want to establish some restrictions to ensure that groups contain both boys and girls as well as racial and ethnic diversity.

Our Choice of Problems

By choosing problems from a variety of sources, we have tried to make students aware of the connections between mathematics and ordinary life activities. Although most students recognize the role of mathematics as a valuable tool for the scientist and engineer, they seldom realize the role that mathematics plays in sports, travel, music, art, money, household activities such as cooking and buying paint and lumber, making everyday estimates, and so on.

We hope that the questions found between these covers will lead students to realize that mathematics is valuable in solving real problems that arise during the course of most lives—a realization that may motivate them to pursue mathematics with greater intensity. Further motivation will come from doing the analyses required by the problems, comparing solutions, and questioning one another as to whether or not the answers seem reasonable.

As you can see from the table of contents, we have chosen a variety of sources to show how math is used in everyday life or in answering interesting questions. As former teachers, we know how difficult it is to find time to write questions or develop problem situations that will both challenge and motivate students. We hope you will find that many of these problems meet your needs, are appropriate for your students, and will help them to see that mathematics can and will play a significant role in their lives.

Name ______________________________ Date ____________________

16. What's My Fielding Percentage?

Baseball and softball players are usually interested in measuring how they are performing. For professionals, performance may be the deciding factor in salary negotiations. Many different statistics are kept by baseball fans and players. Some are cumulative—they just add up—like how many Runs Batted In (RBIs) they have or how many errors. Other statistics are the result of division. Examples of these statistics include Batting Averages, Earned Run Averages (ERAs) and Fielding Percentages.

Look at some statistics for former major league shortstop Rabbit Maranville. In his 23-year career, he managed 5139 assists (throws resulting in an out) and 7354 putouts (catches resulting in an out). In all, he made 631 errors. You can determine his "fielding percentage" by using your calculator and the formula:

(Assists + Putouts) ÷ (Errors + Assists + Putouts) = Fielding Percentage

In Rabbit Maranville's case, the formula above would give us:

(5139 + 7354) ÷ (631 + 5139 + 7354) = 12493 ÷ 13124 = .9519201463.

Like batting averages, fielding percentages are usually rounded off to three figures. Maranville's career fielding percentage then, is given as .952 in most baseball statistic books. Baseball players and fans understand that all fielding percentages are less than 1.000, so they often don't mention the decimal point, saying "his fielding percentage was 952."

With a phrase like "fielding percentage," you might expect this number to be shown as a percent, maybe 95%. But fielding percentages are actually expressed as decimals to three places.

Another way to express a fielding percentage is errorless chances per total chances. Whenever you see the word "per" in a problem requiring math for its solution, it means you are going to have to divide one number by another. If someone tells you that gasoline is selling for $1.50 per gallon, or $1.50 ÷ 1 gallon, each gallon costs $1.50. If a baseball player has 495 successful fielding chances per 500 total chances, his or her errorless chances per total chances is 495 ÷ 500 = .990, or a fielding percentage of 990. Another way of saying this is that the player can be expected to make an error about once in every 100 chances, because 99 ÷ 100 = .990.

(continued)

Name ______________________________ Date ____________________

16. What's My Fielding Percentage? *(continued)*

If you are working in groups, divide into pairs. Each individual can then determine his or her fielding percentage. After your partner checks your calculation, do questions 2, 3, and 4. Then gather in groups of four to compare your answers and work out any differences you may have.

Once you reach agreement on the answers to questions 1–4, again break up into pairs to work on questions 5–10. Then meet once more in groups of four to compare answers and work out any differences.

1. If you are a softball or baseball player, calculate your current fielding percentage or your fielding percentage from last season. Remember that simply catching the ball does not count, only plays that result in outs or errors. If you are not a player yourself, consider the career statistics of Willie Mays as a center fielder (below), and calculate his fielding percentage:

Putouts	Assists	Errors
3328	117	82

Fielding percentage: ______________________________

2. In 1971, right fielder Roberto Clemente played his next-to-last season for the Pittsburgh Pirates before his death in an airplane accident. That year he managed 267 putouts, 11 assists, and made only 4 errors. What was his fielding percentage that year? ______________________

3. Also in 1971, Brooks Robinson of the Baltimore Orioles, often considered the best fielding third basemen of all time, had 131 putouts, 354 assists, and 16 errors. What was his fielding percentage in 1971? ____________

4. Compare the fielding records of Clemente and Robinson for 1971. Explain why you think that fielding percentages are not *all* that is involved in determining the finest fielders in the game. ________________

____

____

(continued)

Name ______________________________ Date ______________________

16. What's My Fielding Percentage? *(continued)*

5. Compare the career fielding percentages of two famous catchers:

   Roy Campanella of the Brooklyn Dodgers (1948–1957) –
   6520 putouts, 550 assists, and 85 errors.
   Fielding percentage: ______________________

   Johnny Bench of the Cincinnati Reds (1967–1983) –
   9260 putouts, 850 assists, and 97 errors.
   Fielding percentage: ______________________

6. Kiki Cuyler, a center fielder for several clubs in the 1920's and 1930's had 4034 putouts, 191 assists, and 121 errors in his career. What was his fielding percentage?

7. In 1996, Alex Rodriguez of the Seattle Mariners won the American League batting championship with a .358 average. During the season he also had 239 putouts, 403 assists, and 15 errors. What was his fielding percentage?

8. In the same year, Rodriguez's teammate, Ken Griffey, Jr., had 374 putouts, 10 assists, and 4 errors. What was his fielding percentage? __________

9. In 1996, Chicago Cubs led the National League in fielding. During the season they had 4369 putouts, 1797 assists, and 104 errors. What was the Cubs' team fielding percentage? ______________________

10. Hall of Famer Ty Cobb, who played major league baseball for 24 years in the early part of the 20th Century, holds the best career batting average in history at .366. His defense was not as good. He garnered 6362 putouts, 392 assists, and 271 errors in all. What was his career fielding percentage?

    ____

Name ____ Date ___________________________

17. What's My Earned Run Average?

Pitchers are statistics followers too. One of the most commonly reported pitching statistics is Earned Run Average or ERA. This number is the average number of earned runs—runs scored without the aid of an error—the pitcher has given up per nine-inning game.

For instance, Eppa Rixey, a pitcher from 1912 to 1933, pitched 4494 innings in his major league career. In those innings he surrendered 1572 earned runs. To calculate Rixey's ERA, we must take his number of innings and divide by 9 to determine how many full "games" he pitched. Then take the number of runs he gave up and divide it by the number of full games he pitched.

In Rixey's case we get 4494 ÷ 9 = 499.33333, and 1572 ÷ 499.3333 = 3.15. An earned run average is usually listed with two digits after the decimal point.

If you are working in groups, divide into pairs. Each individual should then do the following six problems. When you have finished, compare answers with your partner. Then gather again in groups of four to compare your answers and work out any differences you may have.

1. Fielding percentages range from 0.000 to 1.000. Is there a similar range for earned run averages? ____

2. After the 1919 season the American League champions, the Chicago White Sox, were heavily favored to win the World Series. They lost the Series. It was later learned that some of the players had conspired to lose on purpose. They were thrown out of baseball for life. One of these infamous "Chicago Black Sox" was pitcher Eddie Cicotte. In his career, he pitched 3224 innings and gave up 848 earned runs. What was his career ERA?

(continued)

Name ______________________________ Date ____________________

17. What's My Earned Run Average? *(continued)*

3. Firpo Marberry was a pitcher for the Washington Senators, Detroit Tigers, and New York Giants in the 1920's and 1930's. He pitched 2066 innings and gave up 834 earned runs. What was his career ERA?

   ____

4. Hal Newhouser of the Detroit Tigers was the only pitcher to win the Most Valuable Player Award in back-to-back seasons, in 1944 and 1945. During those two seasons he pitched 625 innings and gave up 140 earned runs. What was his MVP ERA?

   ____

5. Sandy Koufax and Bob Gibson were two of the best National League pitchers of the 1960's. In Koufax's 12-year career, before succumbing to arthritis, he pitched 2325 innings and gave up 713 earned runs. Gibson pitched for 17 years, racking up 3885 innings and yielding 1258 earned runs. Compare the ERA's of these two great hurlers.

   Koufax: ______________________________

   Gibson: ______________________________

6. Louis Walter "Kid" Bauer pitched in one game for the Philadelphia Athletics in 1918. In his one appearance in a major league game, he gave up 2 runs without getting any outs. He was therefore credited with an unusual ERA. What was it?

   ____

Name ______________________________ Date ____________________

18. In the Sports Pages

There's a lot of math in the sports pages of your newspaper. Below is one example. You can find many more by reading about your favorite sports in newspapers, magazines, and books.

The table below shows the standings at one point in time for the Eastern Division of the Canadian Football League (CFL). The column headings W, L, T, F, A, and Pts stand for games Won, games Lost, games Tied, points scored For (by) the team, points scored Against the team, and Points acquired by the team thus far in the season.

CFL Eastern Division

Team	**W**	**L**	**T**	**F**	**A**	**Pts**
Hamilton	3	0	0	96	60	6
Toronto	2	1	0	76	74	4
Montreal	1	1	1	42	50	3
Ottawa	0	2	1	42	72	1

If you are working in groups, divide into pairs. Each individual should then do all the problems. When you have finished, compare answers with your partner. Then gather again in the larger group to compare answers and work out any differences you may have.

1. At the time of the standings shown in the table, what percentage of the games it had played had each of the following teams won?
   a. Hamilton: ______________________________
   b. Toronto: ______________________________
   c. Montreal: ______________________________
   d. Ottawa: ______________________________

(continued)

Name ______________________________ Date ____________________

18. In the Sports Pages *(continued)*

2. At the time of the standings shown, which team would you think had:
   a. the best offense: ______________________________
   b. the best defense: ______________________________
   c. Explain your answers to a. and b. above.

   ____

   ____

   ____

3. Using the standings, how many points does a team receive for each:
   a. win: ______________________________
   b. tie: ______________________________
   c. loss:______________________________

4. What was the average number of points scored per game by Hamilton after playing three games? ______________________________

5. What was the average number of points per game allowed by Montreal's defense after playing 3 games? ______________________________

6. Should the total for column F (points scored by all teams) equal the total for column A (points scored against all teams)? ____________________

   Add the two columns. Are they equal? ____________________

Name ____ Date ___________________________

19. More in the Sports Pages

This table gives the standings for American League Teams at the all-star break in July 1996. In the table, the W column gives the games won by each team listed; the L column gives the games lost by each team; the Pct. column gives the decimal fraction of games won for all games played; the GB (games behind) column indicates the number of games that each team trails the team in first place. The American League has three divisions—East, Central, and West. On July 10, 1996, New York led the East, Cleveland was in first place in the Central division, and the Texas Rangers led the West.

American League Teams: July 10, 1996

East	**W**	**L**	**Pct**	**GB**
New York	52	33	.612	___
Baltimore	46	39	____	6
Toronto	38	49	____	15
Boston	36	49	____	___
Detroit	27	61	____	___
Central	**W**	**L**	**Pct**	**GB**
Cleveland	52	35	___	___
Chicago	50	37	___	___
Milwaukee	43	43	___	$8\frac{1}{2}$
Minnesota	41	45	___	___
West	**W**	**L**	**Pct**	**GB**
Texas	51	36	___	___
Seattle	46	39	___	___
California	43	45	___	___
Oakland	43	45	___	$8\frac{1}{2}$

(continued)

Name ______________________________ Date ____________________

19. More in the Sports Pages *(continued)*

If you are working in groups, divide into pairs. You should work out the Pct. for each team in one or more of the three divisions (East, Central, or West). Your partner should do the same. Then compare answers with your partner before the entire group meets to compare results for the entire league. Work out any differences you may have. Repeat the procedure for question 2.

1. Fill in the blanks in the Pct. column.

2. The GB (games behind) column indicates the combined number of games that the trailing team must win and the first-place team must lose in order for the trailing team to catch up to the leading team. It can be calculated as follows:

$$\frac{(\text{lead team's wins} - \text{trailing team's wins}) + (\text{trailing team's losses} - \text{lead team's losses})}{2}$$

For example, Seattle was 4 games behind Texas:

$$\frac{(51-46) + (39-36)}{2} = \frac{(5 + 3)}{2} = 4$$

If Texas were to lose 4 games and Seattle win 4, the standings would be even.

	W	L	GB
Texas	51	40	__
Seattle	50	39	__

$$\frac{(51-50) + (39-40)}{2} = \frac{[1 + (-1)]}{2} = 0$$

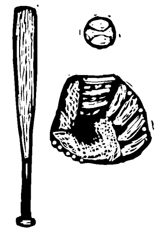
Of course, Texas has played two more games than Seattle (91 vs. 89). If this were the end of the season, the outcome would depend on what Seattle did in its last two games. It could win one and lose one and tie Texas, lose both and lose the division title, or win 2 and win the title.

3. Return to the table of league standings and determine the GB (games behind the first-place team) for each team where that calculation has not been done.

Name ____ Date ___________________________

20. Still More from the Sports Pages

This table gives attendance figures for American League Baseball teams from opening day to the all-star break in July, 1996. This is about half the baseball season. The table gives the total number of games (dates) played at home, the total number of fans who attended these games, and the average number of people attending each game (number per game) for Baltimore and Boston.



Attendance Figures for the first half of the 1996 American League Baseball season.

Team	Dates	Total attendance	Average attendance
Baltimore	46	2,038,534	44,316
Boston	41	1,133,451	27,645
California	44	1,054,715	_____
Chicago	40	843,836	_____
Cleveland	44	1,839,190	_____
Detroit	43	638,789	_____
Kansas City	43	781,556	_____
Milwaukee	40	585,510	_____
Minnesota	44	766,877	_____
New York	45	1,119,291	_____
Oakland	41	516,942	_____
Seattle	44	1,455,177	_____
Texas	46	1,610,768	_____
Toronto	41	1,289,964	________
A.L. Totals	___	________	________
N.L. Totals	601	16,009,881	26,639

(continued)

Name ______________________________ Date ____________________

20. Still More from the Sports Pages *(continued)*

If you are working in groups, check your individual answers to question 1. Then divide up the calculations for question 2 so that each individual has to do no more than six. Be sure that at least two people do each team so they can check each other's answers.

Once there is agreement about question 2, divide into pairs to answer question 3. One member of the pair can read the numbers while the second uses a calculator to add the figures. Compare results in your larger groups and work out any differences before discussing the answers to question 4 as a large group.

1. Check the average attendance figures for Baltimore and Boston. Do the figures in the tables agree with your calculations? ____________________

\ 2. Fill in the average attendance column by calculating the average attendance figures for the other American League teams (California to Toronto).

3. Find the total games (dates) played by all American League teams for the first half of the season. Then fill in the blank beside A. L. (American League) Totals. Do the same for total attendance. Then calculate the league's average attendance figure and fill in that blank.

4. How do the totals for dates and attendance for the American League compare with those given for the National League?

____

____

____

____

____