Word Problems with Fractions

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To the Teacher

Here's a book designed to build student skills in two areas of concern for most teachers: working with fractions and applying arithmetic skills through word problems.

Word Problems with Fractions parallels two other books by Paul R. Robbins and Sharon K. Hauge, *Word Problems with Whole Numbers* and *Word Problems with Decimals, Proportions and Percents*. The books are not dependent on one another, but they can be used together very well.

This series of books came into existence at the urging of J. Weston Walch, the founder of J. Weston Walch, Publisher. Mr. Walch recognized the need to provide materials for students that would not only teach the fundamentals of arithmetic but would also show the students how to use these skills to solve word problems. He wanted texts that would help students learn these skills while keeping the students engaged and interested in learning.

The three books for solving word problems emerged as a way of meeting this challenging task. We were very pleased that the approach we used won acceptance by many classroom teachers as a tool for teaching problem-solving skills to their students. The books have remained staples in the Walch catalog since their publication. Their reproducible format has kept them perenniel favorites among veteran teachers and those new to the field.

It is now time for a new edition of these texts. There are a number of reasons for this decision. One reason has to do with the prices mentioned in many of the word problems in the texts. To keep the word problems credible to the students, we had to use new, realistic prices. A second reason for the new edition is the explosion of new technologies that entered the lives of young people. We wanted to include word problems that used these technologies as well as new information and ideas that have come from science. Third, many students are now using hand-held calculators. We believe it is important to show students how to use calculators as tools for solving word problems.

With these needs in mind, we offer the second edition of *Word Problems with Fractions*.

Word Problems with Fractions is written at a level which almost all middle school students will handle comfortably. It presents a series of problems that young people will find not only interesting but worth trying to solve. And it presents the subject of fractions with rare lucidity and conciseness.

We hope that you will continue to find *Word Problems with Fractions* useful in your classroom teaching. We welcome your comments.

To the Student

Word Problems with Fractions is the second volume in our series of books that explain how to solve word problems in basic mathematics. In the other books, we cover word problems that use whole numbers and decimals, proportions, and percents. In this text, we shall show you how to solve problems that use fractions.

In the earlier book, *Word Problems with Whole Numbers*, we explained in detail how to recognize which operations of arithmetic are needed to solve a particular problem; that is, whether you need to add, subtract, multiply or divide to find the correct answer. We did this by pointing out certain key words, phrases or ideas that are presented in the problem that act as signals or guides to help you decide what to do. In this text, we shall continue in this manner. However, we shall be briefer, since we have discussed some of these ideas before.

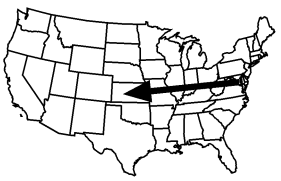
Name _____

Date _____

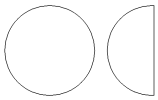
Reproducible

Introduction to Fractions

This material is all about fractions. We will explain how to add, subtract, multiply, and divide fractions and how to use these techniques to solve word problems. We will also talk about reducing fractions and working with mixed numbers. Because many people have trouble with fractions, we will start off very carefully and try to present a clear idea of just what fractions are and what they mean. To do this, we will begin with some things we all know about from everyday experience. For example, let's start with the idea of one half ($\frac{1}{2}$). What does $\frac{1}{2}$ mean?



There are a lot of ways we use one half. One meaning is "halfway." If you were driving from Washington, D.C. to California, you would be **halfway there** when you reached Kansas.



Think of a full moon—nice, round, and yellow.

Now think of half a moon.

Think of a bottle of cola. Drink half of it. What's left?

Or look at the people in this picture. One-half are males and one-half are females.

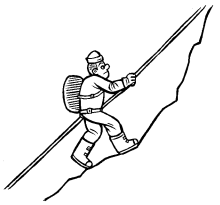
Now let's talk about one quarter ($\frac{1}{4}$). Instead of a half ($\frac{1}{2}$) dollar, we have:

Or we could drink more of the cola:

We can also talk about a quarter moon:

or one quarter of the way across the country.

You can divide things into any number of equal parts. For example:



This young man has climbed about two thirds ($\frac{2}{3}$) of the way up to the top of the hill.

A dime is one tenth ($\frac{1}{10}$) of a dollar.

My gasoline tank is three quarters ($\frac{3}{4}$) full. My stock was down seven eights ($\frac{7}{8}$) of a point.

If you live in the United States, you are one two-hundred-sixty-five-millionth ($\frac{1}{265,000,000}$) of the population of the United States.

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The Numerator and the Denominator

By now, you have undoubtedly discovered that a fraction has two parts. ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{2}{3}$, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{265{,}000{,}000}$)

You write a fraction by putting one number above the line and one number below the line. The number above the line is called the **numerator**. The number below the line is called the **denominator**.

$\frac{2}{3}$ $\frac{\text{Numerator}}{\text{Denominator}}$

The numerator can be any whole number and the denominator can be any whole number, except zero.

Sometimes the numerator and the denominator are called the **terms of the fraction**.

A Guide to Reading Fractions

1. First, read the whole number in the numerator.

Then, read the whole number in the denominator,

(a) adding "-th" if the numerator equals 1

(b) or adding "-ths" if the numerator is greater than 1.

Examples: $\frac{1}{6}$ one six**th**; $\frac{5}{6}$ five six**ths**

While the above rule is helpful, there are a number of exceptions to it. Here are five common exceptions.

(a) half: $\frac{1}{2}$ one half
halves: $\frac{3}{2}$ three halves

(b) third: $\frac{1}{3}$ one third
thirds: $\frac{2}{3}$ two thirds

(c) fifth: $\frac{1}{5}$ one fifth
fifths: $\frac{2}{5}$ two fifths

(d) first: $\frac{1}{21}$ one twenty-first
firsts: $\frac{3}{21}$ three twenty-firsts

(e) secondth: $\frac{1}{32}$ one thirty-secondth
secondths: $\frac{3}{32}$ three thirty-secondths

2. If the number in the denominator ends in "y", change "y" to "ie" before adding **th** or **ths**.

Example: $\frac{1}{40}$ one fortieth

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Uses of Fractions

While a common fraction has only one mathematical definition, one can look at the ways fractions are used in problems from somewhat different viewpoints. Here are three of the ways fractions are used.

A. Fractions can be used to show that a whole thing or unit has been broken up into a certain number of equal parts and that we have a special interest in some of these parts.

B. Fractions can be used to represent one or more equal parts of a total collection of units.

C. Fractions can be used to compare two quantities.

Let's try to make these ideas clearer. Let's begin with the first one. We will look at the others in later lessons.

You can break up almost anything into equal parts. Take a football field. It is divided into one hundred equal yards. Let's suppose your team is on your opponent's one-yard line. You have only one more yard to go and you score. What part is that one yard of the football field? If you said $\frac{1}{100}$ you are right.

A Handy Rule for Writing Fractions (A)

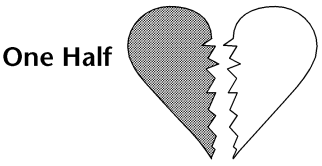
To write a fraction which shows that one whole thing has been broken into a certain number of equal parts, and that we have a special interest in some of these parts:

1. First, **ASK** how many equal parts the whole thing has been broken into.
2. **WRITE** this number and **DRAW** a line above it.
3. Next, **ASK** in how many of these parts do we have a special interest?
4. **WRITE** this number above the fraction line.

The following examples illustrate this idea.

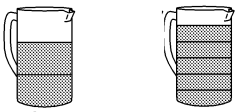
Something has been broken into 2 equal parts.

I have one of these parts.



1. How many equal parts has the whole heart been broken into? 2
2. $\frac{}{2}$
3. How many of these parts do we have special interest in? 1
4. $\frac{1}{2}$

Something has been broken into 3 equal parts.
I have two of these parts.
Something has been broken into 6 equal parts.
I have five of these parts.



$\frac{2}{3}$ $\frac{5}{6}$

Two Thirds Five Sixths

Example:	A house has been divided into 4 equal rooms. What fraction of the house is one room?
Solution:	The whole thing is the house. It has been divided into 4 equal rooms. Hence, 4 should be written below the line or in the denominator. You are particularly concerned with 1 of these rooms, or **one** of the equal parts. Therefore 1 should be written above the line or in the numerator. The answer is $\frac{1}{4}$.

Is this idea clear? Try these exercises and see.

Word Problems for Practice (I)

1. A week is divided into 7 days. What fraction of a week is 1 day?
2. A chocolate bar was divided into 8 equal parts. What fraction of the bar are 5 of these parts?
3. The design of a space shuttle for a future space exploration called for the shuttle to be divided into 3 equal sections. The first section would be the living quarters for the men and women who would navigate the shuttle. The second section was to be used for scientific studies. The third section would be storage room for supplies. What fraction of the space shuttle would be used for living quarters?
4. A football game was divided into 4 periods of equal length. After the first period of the game was over, what fraction of the game had been completed?
5. Beth and Keisha invited their boyfriends out for a pizza. The pizza was divided into 8 equal pieces. After the 4 friends had each eaten 1 piece, what fraction of the pizza had been eaten?
6. A student had a part-time job. He received a take-home pay check of $120 each month. He split the money up equally into 3 parts, using the money to pay for snacks, compact discs, and to take out his girlfriend. What fraction of his pay check did he use for snacks?

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