Word Problems with Decimals, Proportions, and Percents

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J. WESTON
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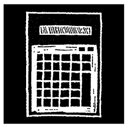

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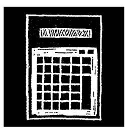


To the Teacher

This book of reproducible instructional text is designed to build student skills in several areas of major concern for most teachers: decimals, proportions, percents, and application of arithmetic skills through word problems.

Word Problems with Decimals, Proportions, and Percents parallels two other books by Paul R. Robbins and Sharon K. Hauge, *Word Problems with Fractions* and *Word Problems with Whole Numbers*. The books are not dependent on one another, but the current volume does, of necessity, assume student knowledge of fractions.

This series of books came into existence at the urging of J. Weston Walch, the founder of J. Weston Walch, Publisher. Mr. Walch recognized the need to provide texts for students that would not only teach the fundamentals of arithmetic, but would also show the students how to use these skills to solve word problems. He wanted texts that would help students learn these skills, while keeping the students engaged and interested.

The three word problems books emerged as a way of meeting this challenge. We were very pleased that the approach we used won acceptance by many classroom teachers as a tool for teaching problem-solving skills to their students. The books have remained staples in the Walch catalog since their original publication in 1982.

It is now time for a new edition of these texts. There are a number of reasons for this decision. One reason has to do with the prices mentioned in many of the word problems. To keep the word problems credible to the students, we had to use new, realistic prices. A second reason for the new edition is the explosion of new technologies that has entered the lives of young people. We wanted to include word problems that used these technologies as well as new information and ideas that have come from science. Thirdly, many students are now using hand-held calculators. We believe it is important to show students how to use calculators as tools in solving word problems.

With these needs in mind, we offer the second edition of *Word Problems with Decimals, Proportions, and Percent*.

Like the other two books in this series, *Word Problems with Decimals, Proportions, and Percents* is written at a level which almost all junior high and even younger students will handle comfortably. What's more, it can be used to advantage in consumer and general math classes at the high-school level. It presents a series of problems that young people will find not only interesting but worth trying to solve. And it presents the topics of its title with rare lucidity and conciseness.

We hope that you will continue to find *Word Problems with Decimals, Proportions, and Percents* useful in your classroom teaching. We welcome your comments.

To the Student

Word Problems with Decimals, Proportions, and Percents is the third volume in our series of worktexts that explain how to solve word problems in basic mathematics. In our earlier texts, we covered word problems that use whole numbers and fractions. In this text, we shall show you how to solve problems that use decimals, proportions, and percents.

In the earlier books, we explained in detail how to recognize which operations of arithmetic are needed to solve a particular problem: that is, whether you need to add, subtract, multiply or divide to find the correct answer. We did this by pointing out certain key words, phrases or ideas that are presented in the problem that act as signals or guides to help you decide what to do. In this text, we shall continue in this manner. However, we shall be briefer, since we have discussed some of these ideas before. If you have not worked with our earlier books, *Word Problems with Whole Numbers* and *Word Problems with Fractions*, we suggest that you look through them, as this will help you more fully understand the approach we will take in solving word problems in this workbook.

Name ____________________

Date ____________________

Reproducible

Dividing with Decimals

Adding and subtracting decimals is quite simple. Multiplying decimal fractions is a little harder. Dividing with decimal fractions is a little harder still. But not all that hard. If you can divide whole numbers, you can divide decimal fractions. You do the same thing. The only thing you have to worry about is **where to put the decimal point**. Put it in the wrong place and you are out of luck.

Before we begin, let us briefly review the meaning of three very important words: **divisor**, **dividend**, and **quotient**.

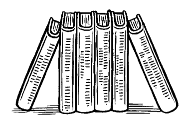
When we divide:

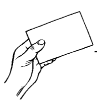
1. The number we divide by is the **divisor**.
2. The number that is divided is the **dividend**.
3. Our answer is the **quotient**.

Example

```
                      5   <----  Quotient
Divisor  ---->   4 ) 20   <----  Dividend
```

Now, there are three cases to learn about dividing with decimal fractions. Let us go to our mathematics bookshelf and pick up Case 1, "Decimal in the Dividend, But Not in the Divisor."





A Handy Rule for Dividing Decimals (A)

CASE 1: A Decimal in the Dividend, But Not in the Divisor.

This is where your divisor has no decimal point (like 65 in the example below), but your dividend has a decimal point (like 81.25 in the example below).

In this case,you divide as usual and put the decimal point in your answer **right above** the decimal point in your dividend.

Like so:

```
       1.25
   65 )81.25
       65
       ----
       16 2
       13 0
       ----
        32 5
        32 5
       ----
```

(continued)

Other examples:

Examples	$\begin{array}{r} 4.2 \\ 3\overline{)\,12.6} \end{array}$	or	$\begin{array}{r} 140.16 \\ 6\overline{)\,840.96} \end{array}$

Drill for Skill (VIII)

You can check each answer by multiplying the divisor by the answer. If your work is correct, the product will give you the dividend.

1. $3\overline{)96.27}$ Check:

2. $5\overline{)915.30}$ Check:

Division problems like the following example often cause students difficulty. Please look at this example carefully.

$14\overline{).28}$	Place the decimal point in the quotient.
$\begin{array}{r} . \\ 14\overline{).28} \end{array}$	But notice that 14 will **not** go into 2, because it is larger than 2. To show this, you must write 0 in the quotient.
$\begin{array}{r} .0 \\ 14\overline{).28} \end{array}$	Now think 14 will go into 28 twice, so write 2 in the quotient.
$\begin{array}{r} .02 \\ 14\overline{).28} \\ \underline{28} \end{array}$	Your answer is .02.

Drill for Skill (IX)

Here are some problems like the above example for you to try. To be sure that each of your answers is correct, check it.

1. $15\overline{).30}$ Check:

2. $12\overline{)1.08}$ Check:

(continued)

3. $32\overline{).064}$ Check:

4. $49\overline{).00147}$ Check:

You may have noticed that in each of the previous problems, the quotient came out exactly—there was no remainder. You might have asked, "What if the division does not come out exactly?" Well, there are two things that might happen. Let's take a look at each of them.

1. In the first case, the division finally comes out exactly if we add a zero or zeros at the end of the dividend. Here is an example:

$$\begin{array}{r} .25 \\ 25\overline{)6.35} \\ \underline{5\,0} \\ 1\,35 \\ \underline{1\,25} \\ 10 \end{array}$$

We have used all the digits in the dividend, but the division did not come out exactly. However, you will recall from our earlier work that 6.35 is the same as 6.350. So, write a zero at the end of 6.35, which will make the number 6.350. Continue to divide.

$$\begin{array}{r} .254 \\ 25\overline{)6.350} \\ \underline{5\,0} \\ 1\,35 \\ \underline{1\,25} \\ 100 \\ \underline{100} \end{array}$$

The division has ended.

2. In some problems, our division process never stops—regardless of how many zeros we write on the tail of the dividend. Here is an example of this:

$$\begin{array}{r} .3 \\ 3\overline{)1.0} \end{array} \qquad \begin{array}{r} .33 \\ 3\overline{)1.00} \\ \underline{9} \\ 10 \\ \underline{9} \\ 1 \end{array} \qquad \begin{array}{r} .333 \\ 3\overline{)1.000} \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

You can see that the division never stops!

(continued)

You might ask, "If the division never stops, how do I ever write an answer for problems like this?" Well, the answer you write down depends upon how many decimal places you want to include in your answer.

(a) If you want one decimal place in your answer, carry out the quotient to two decimal places. Then, follow the rules for rounding and round your answer off to one decimal place.

(b) If you want two decimal places in your answer (if you're dealing with dollars and cents, for instance), carry out the quotient to three decimal places. Then, round your answer off to two decimal places.

(c) If you want three decimal places in your answer (if you're dealing with batting averages, for instance), carry out the quotient to four decimal places. Then, round your answer off to three decimal places.

- The quotient in the above problem rounded to one decimal place is .3.
- The quotient in that same problem rounded to two decimal places is .33.

We could go on and on and on.

Before we bring down Case 2 from our bookshelf on dividing with decimals, this seems like a good place to answer a question we posed earlier.

Do you remember when we promised to show you how to change a common fraction into a decimal? Now you're ready for the explanation.

A Handy Rule for Changing a Common Fraction into a Decimal

To change a fraction into a decimal simply divide the bottom into the top.

Example

$\frac{1}{4} = 4\overline{)1}$

4 will not go into 1, but 1 = 1.0

$$\begin{array}{r} .2 \\ 4\overline{)1.0} \\ 8 \\ \hline 2 \end{array}$$

Add another 0 to the dividend:

$$\begin{array}{r} .25 \\ 4\overline{)1.00} \\ 8 \\ \hline 20 \\ 20 \\ \hline \end{array}$$

So $\frac{1}{4}$ is the same as the decimal fraction .25.

This makes good sense, because $.25 = \frac{25}{100} = \frac{1}{4}$.

(continued)

Name ______________________________

Date ______________________________

Example

$$
\begin{array}{r}
.333 \\
\tfrac{1}{3} = 3 \overline{\smash{)}\,1.000} \\
9 \\
\hline
10 \\
9 \\
\hline
10 \\
9 \\
\hline
1
\end{array}
$$

In this situation, as we discussed earlier, the division process does not stop—we get .333333333333333 . . . (The " . . ." is a sign which tells us that we could go on forever and ever and ever.) So $\frac{1}{3}$ cannot be written as a decimal fraction, but it can be approximated (i.e., we can find an answer that is not exact, but close) by lots of decimal fractions, .3 or .33 or .333 or .3333 or . . ., depending upon how many decimal places we want to use.

Drill for Skill (X)

1. Change each of the following common fractions into a decimal fraction:

(a) $\frac{1}{5}$ = ____________ (d) $\frac{1}{8}$ = ____________

(b) $\frac{2}{5}$ = ____________ (e) $\frac{7}{8}$ = ____________

(c) $\frac{3}{4}$ = ____________ (f) $\frac{3}{10}$ = ____________

2. Change each of the following common fractions to a 2-place decimal fraction. Your answer will be only an approximation. (Carry out the division to **3** decimal places and follow the rules for rounding to round off your answer to two decimal places.)

(a) $\frac{2}{3}$ ~ ____________ (b) $\frac{1}{6}$ ~ ____________ (c) $\frac{1}{16}$ ~ ____________

The symbol ~ means **approximately**.

Now let's haul down Case 2 from our bookshelf.

A Handy Rule for Dividing Decimals (B)

CASE 2: A Decimal in the Divisor, But Not in the Dividend.

This is where your divisor contains a decimal point, such as 2.5, but your dividend does not, such as 100.

In this case, (a) Forget about the decimal point in the divisor.
(b) Add as many zeros to the dividend as there were decimal places in the divisor.
(c) Now, divide.

(continued)

Name ______________________________ Date ______________________________

Reproducible

Dividing with Decimals

Example $2.5\overline{)100}$

The divisor 2.5 has one decimal place.

The dividend does not have a decimal point.

Follow the rule and write

$$\begin{array}{r} 40 \\ 25\overline{)1000} \\ 100 \\ \hline 0 \\ 0 \\ \hline \end{array}$$

If your problem were $.25\overline{)100}$, you would write $25\overline{)10000}$.

If your problem were $.025\overline{)100}$, you would write $025\overline{)100000}$, which is the same as $25\overline{)100000}$.

Drill for Skill (XI)

In these problems, carry out the division two decimal places and round your answer to one decimal place.

1. $26.4\overline{)658}$ __________
2. $3.14\overline{)593}$ __________
3. $1.4\overline{)562}$ __________
4. $2.6\overline{)24}$ __________

NOTE: If the division does not come out exactly, your rounded-off answer multiplied by the divisor will not give the dividend exactly—EVEN IF ALL OF YOUR WORK IS CORRECT. However, if your work is correct, your rounded-off answer multiplied by the divisor should be close to the dividend.

Now, are we ready for Case 3.

A Handy Rule for Dividing Decimals (C)

CASE 3: Decimals in Both the Divisor and the Dividend.

Here's what you do. $.12\overline{)10.80}$

Count the number of decimal places in the divisor. Move the decimal point in both the divisor and the dividend to the RIGHT the number of places you counted.

(continued)

Example	In the example $.12\overline{)10.80}$, the divisor, .12, has 2 decimal places, so move the decimal point in both the divisor and dividend 2 places right. This gives you $12\overline{)1080.}$ $\begin{array}{r} 90 \\ 12\overline{)1080.} \end{array}$ If you had $1.936\overline{)96.8}$, you would move your decimal points 3 places to the right in both divisor and dividend. To do this, you would have to add 2 zeros in the dividend to give you enough places to move the decimal. You would then have $1936\overline{)96800.}$

Drill for Skill (XII)

Try these. Carry out each division until the division process stops.

1. $.8\overline{)36.81}$ __________
2. $.42\overline{)8.4}$ __________
3. $2.4\overline{)96.96}$ __________
4. $1.3\overline{)26.39}$ __________

Solving Word Problems That Require Division of Decimals

We shall discuss several types of word problems that you can solve using your skills of dividing with decimals.

Type I

The first type of problem occurs when you are asked to divide something into equal parts. For example, imagine you have a board 3.6 meters long and are asked to divide it into 6 equal parts. How long would each part be?

Type II

The second type of problem asks you how many things of equal size will fit into a larger-sized space or container. For example, suppose you have a shelf that is 2.5 meters long and are asked how many .5-meter-long boxes will fit on the shelf.

Type III

The third type of problem deals with division problems which contain the word **per**. We will show you how to tell the difference between these problems and the problems using **per** that you solved earlier by multiplication.

Let's begin with the first type of problem mentioned.

(continued)

Name ______________________________

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Reproducible

Dividing with Decimals

Type I: Division of a Whole into Equal Parts

To solve these problems, follow this simple rule.

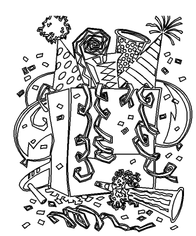
A Handy Rule for Division Word Problems (A)

1. Put whatever it is you are dividing up on the **left** side of the division sign: ↑ ÷
2. Put the number of things you are dividing it up into on the right side of the division sign: ÷ ↑
3. Divide.

Example	Mrs. Mahoney received a check in the mail for $60.90. She decided to divide the money equally into two parts; one part to pay household expenses, the other to buy presents for her grandchildren. How much money would she have for the presents? ___________
Solution	Ask: What is she dividing up? It's money, $60.90. Put this to the left of the division sign: 60.90 ÷
	Ask: How many parts is she dividing the money into? 2 parts. Put this to the right of the division sign: 60.90 ÷ 2
	Divide: $2\overline{)60.90}$ = 30.45 The answer is $30.45.

Word Problems for Practice (VIII)

1. Wendell built a small greenhouse to put on the back porch of the house. The greenhouse was 1.8 meters long. He decided to divide the greenhouse into 3 equal parts in which to grow different types of plants. How long would each section be? ___________
2. Four candidates were running for mayor. The public television station set up a program where the four candidates would debate each other. When the program came toward the end, there were 2.8 minutes left. The announcer then divided this remaining time equally for a closing statement by each candidate. How much time would each candidate have? __________
3. On New Year's Eve, the people living at 193 Perry Street threw a big party with many guests. The next day, they had to split up a bill for $132.60. One of the four hosts, Jim, offered to pay the sixty cents, but this did not go over too well. The four decided to split up the bill evenly. How much did each pay? ___________



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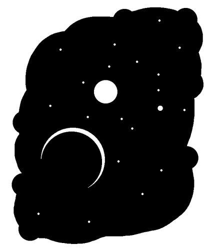
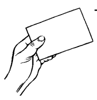
4. Two college roommates decided to split equally the cost of getting on the Internet. They decided to use a local online service which charged $199.96 for a year. How much did each roommate have to pay? ___________

5. At the mountain observatory, time using the powerful new telescope was limited. The program director announced that 15.9 hours of viewing time would soon be made available. Three staff astronomers asked for equal shares of this time. How many hours would each astronomer have for viewing? ___________



Type II: Fitting Equal Parts into a Whole

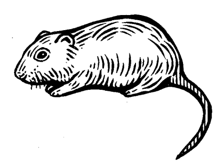


A Handy Rule for Division Problems (B) That Ask You to Find the Number of Things of Equal Size or Amount That Can Fit into a Larger Container

To solve problems like this, there is an easy rule:

Divide the larger thing by the smaller thing.

Example: The pet store had a number of hamsters which were kept in small cages. If the cages were each .3 meter wide, how many of the cages could fit into a 2.1-meter-wide space? ___________



Solution: To solve problems like this, divide the larger thing by the smaller thing.

Larger thing	Smaller thing
2.1-meter space	.3-meter size for each cage
2.1 ÷ .3	
.3 ⟌ 2.1	(Move each decimal point one place to the right.)
3 ⟌ 21	Your answer is 7 cages.

Word Problems for Practice (IX)

1. Mrs. Balzanowski has a 24-inch kitchen shelf where she stores different spices to liven up her cooking. The spices, such as garlic powder, clove, and oregano, are kept in small tin boxes, each 1.6 inches in length. How many of these boxes of spices can fit on the shelf? ___________
2. Ashton keeps his collection of jazz albums recorded in the 1950's in a record cabinet he built himself. The cabinet has a space which is 16.4

(continued)

inches long to hold record albums. How many record albums, each measuring .2 of an inch, could fit into this space? ___________

3. A track for auto racing was shaped like an oval. Each time a car went around the track, it covered 1.2 kilometers. How many trips would a car have to make around the track to cover a distance of 480 kilometers? ___________
4. Wilhelm liked to garden. During the spring, he planted tomato seeds in small boxes and kept them inside the house on a ledge near a sunny window. The ledge was 36 inches long. If each box was 3.6 inches long, how many boxes would fit on the ledge? ___________
5. One of the very early cars, the 1902 Covert, had an engine with only 6.5 horsepower. How many of those engines would you need to equal the power of a modern car with 260 horsepower? ___________

Type III: "Per" Problems That Require Division

In our section on multiplication, we stated that the word **per** meant **each**. We solved problems like "You are driving 60 kilometers per hour and you have driven 2.5 hours. How far have you driven?" You can see that in the multiplication problem you were asked to find a **total** amount, in this case, the total distance traveled.

Per can also be used in division problems. When this happens, you are **not** asked to find a total amount. The total is given to you in the problem. Instead you may be asked to find **the amount per something**, like the number of kilometers per hour or the cost per person. This handy rule will help you.

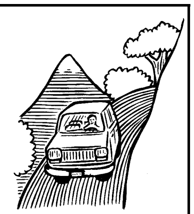
A Handy Rule for "Per" and Division

When you are given a total amount and are asked to find the amount **for each** or **amount per something**, you use **division**.

Turn back to page 33 and compare the rule found there with this new rule.

Example Let's try an example using kilometers per hour. Imagine you have been driving over the mountains. It's a twisting, turning road and you are not going very fast. In fact, after 1.5 hours, you've only gone a total of 16.5 kilometers. How many kilometers per hour have you been traveling?

____



You can see that this problem gives you the total distance traveled (16.5 kilometers). So you know you are not going to have to multiply. What you are asked to find is the number of kilometers traveled per hour. **To find an amount per something, you must divide**.

Solution You may ask, just what do I divide?

Here is a handy way of solving division problems using "per."

A Handy Rule for Division Word Problems (C)

1. Take the basic information you are looking for out of the sentences.

   In this case, you are trying to find KILOMETERS TRAVELED PER HOUR.

2. Remember that what comes before **per** goes before the division sign ↑ ÷

   What comes after **per** goes after the division sign. ÷ ↑

3. Take out the numbers and put them where the words are. In the problem where you are driving the car over the mountain, put 16.5 before the division sign (the total number of kilometers traveled) and 1.5 after the division sign (the number of hours that have gone by).

4. Now, divide: 16.5 ÷ 1.5 $1.5\overline{)16.5}$ Move your decimal point one place to the right in both divisor and dividend.

$$\begin{array}{r} 11 \\ 15\overline{)165} \\ \underline{15} \\ 15 \\ \underline{15} \end{array}$$

   Your answer is 11 kilometers per hour.

Word Problems for Practice (X)

1. Rebecca was doing some volunteer work recording books on tapes for persons who were blind or couldn't see well. During 2.25 hours, Rebecca recorded 45 pages from a new novel. How many pages was that per hour?
   ____
2. T-shirts were on sale at a price of $15.30 per half dozen. What was the cost per T-shirt? ____________
3. The Mudtown softball team had nine players. If it cost $541.62 to provide uniforms and equipment for the players, how much would that be per person? ____________
4. An earthquake destroyed many buildings in Capital City and damaged the reservoir. While the reservoir was being repaired, 3.5 million gallons of water had to be brought into the city each day. How much water would that be per person if 2.5 million people lived in Capital City? ____________
5. Joe was a nurse. The doctor told him to give his patient .75 grams of medication. If the doctor also explained to Joe that there were .5 grams per tablet, how many tablets should Joe give his patient? ____________

(continued)

6. Kay had a pet hermit crab named Hac. One day, Kay let Hac out of the fishbowl where she kept him and let him wander around to get some exercise. During 3 minutes, Hac walked a distance of 2.7 meters. How many meters was that per minute?___________
7. A used bookstore was selling old wildlife and outdoor magazines for 3 for $1.20. How much money would that be per magazine?___________
8. In a science-fiction novel, a spaceship was hurtling toward the stars at a fantastic speed. It traveled 260,000 kilometers in only 2.5 seconds. How many kilometers per second was the spaceship traveling?___________

The Princess with the Purple Hair

Uncle Bill was a great storyteller. During the warm summer evenings, he would sit around with the neighborhood kids and tell them stories. One night, he said to the kids, "Tonight, I'm going to tell you a fairy tale called 'The Princess with the Purple Hair.' It's almost as famous as 'Snow White and the Five Dwarfs.'"

"Seven dwarfs," Junior Hawkins interrupted.

"What's one character more or less?" Uncle Bill said. "Quit nit-picking." He then began his story. "Well, the princess was of course very beautiful, and she had long blonde hair. Actually, I thought about calling her Goldilocks, but there already is such a character in a story about two bears."

"Three bears," Junior Hawkins interrupted.

"Three, you say? Okay. Well, the princess lived in a land where most of the women had blonde hair, and she decided she wanted to be different. She wanted to be noticed. So, she dyed her hair deep purple.

"When the people saw her purple hair, they were all amazed. People came from far and wide to see the pretty princess with the purple hair. Everything seemed just fine until one day, a giant who lived in a castle on a neighboring hill heard about her and decided to steal her away. The giant actually wasn't such a bad fellow until he read a book called *George and the Beanstalk*."

"*Jack and the Beanstalk*," interrupted Junior Hawkins for the third time.

Uncle Bill ignored the interruption and continued.

(continued)

The Princess with the Purple Hair
(continued)

"Well, when the giant read the book, he learned that giants were supposed to be mean creatures who do crummy things like stealing princesses. So, he figured he had to change his image. One night, he stole the princess away, took her to his castle, and then—to be even meaner—he told her that he was going to cut off her purple hair. 'Bit by bit,' he said, as that sounded even meaner. He got out his ruler and found the tresses of her hair measured 12.6 inches long. 'I'm going to cut off your purple hair 2.1 inches each day. And when I come to the end of these tresses, I'm going to turn you into a plum pudding.' Then he let out a feindish-sounding, 'Ho, Ho, Ho!' which nearly scared the poor princess to death.

"Well, as the days went by, the mean giant clipped off the princess's purple tresses piece by piece until the last one was gone. That very evening, the giant was supposed to return to carry out his threat to turn the princess into a plum pudding. But instead of the giant, a handsome young prince in shining armor walked into the room. He had come to rescue her. He beat up the giant and sent him running out of the castle as fast as his legs could carry him.

"Then, the prince took the princess into his arms. After he kissed her, he said, 'I'm sorry I got here too late to save your purple hair.'

"'That's all right,' she replied. 'I've found that having purple hair just gets you into trouble. Besides,' she added, while gazing into the prince's deep blue eyes, 'I think I shall go back to being a blonde. I've heard it said somewhere that blondes have more fun.'"

While the princess was figuring out how she could have more fun as a blonde, can you figure out how many days it took the giant to clip off all the princess's purple locks? ________________