10-Minute Critical-Thinking Activities for Math

by
Hope Martin

J. WESTON
WALCH
PUBLISHER
Portland, Maine

Contents

To the Teacher

What does it mean to *think critically* in the mathematics classroom? Critical thinking is disciplined, self-directed thinking where students are asked to employ previously learned skills and concepts in a problem-solving setting. The problems and activities in *10-Minute Critical-Thinking Activities for Math* have been designed to (1) develop students' reasoning and thinking skills, (2) encourage them to analyze their results, and (3) require students to justify or reexamine their procedures.

However, problems, in and of themselves, cannot be defined as critical-thinking problems. It may be the type of activity that encourages students to reason, analyze, make modifications, and reexamine their procedures. Then we, as teachers, must tempt our students to become actively involved in their learning. These warm-ups support active learning by asking students to explain their reasoning and share their strategies with the class, encouraging them to develop orginal problems and share them with the class, and inviting them to reevaluate their work and check for accuracy.

The activities in *10-Minute Critical-Thinking Activities for Math* contain non-traditional problems designed to encourage students to use critical or logical thinking skills. They have been grouped into five sections:

1. Critical Thinking and Algebra;
2. Logic and Critical Thinking;
3. Number Theory and Problem Solving;
4. Sequences, Patterns, and Codes; and
5. Visual/Geometric Patterns.

In each section the activities are arranged progressively from the easiest to the most difficult. However, do not feel you must complete all of the activities in a section before moving on. Any problem can be used at any time; no sequence of skills is assumed. By selecting problems from different sections, teachers can bring variety into the classroom and revisit topics throughout the year.

Many of the problems in this book are considered open-ended—they may have one correct answer but more than one way to find it, or the problem may actually have more than one correct answer. Open-ended problems encourage students to try different methods of solution and support divergent strategies and a variety of learning styles. When students are encouraged to find multiple solutions using a variety of strategies, not only do they become better problem solvers, but mathematics becomes a creative expression of their talents.

The activities in *10-Minute Critical-Thinking Activities for Math* provide mathematical enrichment for students while also addressing topics that are a meaningful part of the mathematics curriculum. These include the following:

- Number theory
- Computation skills
- Geometry concepts and skills
- Mathematical reasoning
- Sequencing and patterning
- Order of operations
- Algebra concepts and skills
- Spatial visualization and transformations

Some teachers may wish to use these masters to make overhead transparencies; other may wish to give students their own copy.

So use the first 10 minutes of your teaching period to expand your students' mathematical horizons (and knowledge) with these stimulating warm-ups! Remember, 10 minutes a day is 1,800 minutes a year, or about 9 weeks of classes each year! Enjoy mathematics each and every minute!

The Activities

Matrix of Critical-Thinking Skills

Page No.	Activity	Problem Solving	Mathematical Reasoning	Mathematical Communication	Generalization	Visual Thinking	Logic	Patterns & Sequences	Analytic Computation	Analysis	Open-Ended Problems
2	He Said—She Said 1	✔	✔		✔			✔	✔	✔	✔
3	He Said—She Said 2	✔	✔		✔			✔	✔	✔	✔
4	What Comes Next 1	✔	✔		✔	✔		✔	✔	✔	✔
5	What Comes Next 2	✔	✔		✔	✔		✔	✔	✔	✔
6	What Comes Next 3	✔	✔		✔	✔		✔	✔	✔	✔
7	What Comes Next 4	✔			✔	✔		✔	✔	✔	✔
8	The Ladybug Concert	✔	✔	✔			✔		✔	✔	✔
9	The Peanut Problem	✔	✔	✔			✔		✔	✔	✔
10	Number Magic 1	✔	✔	✔			✔		✔		✔
11	Number Magic 2	✔	✔	✔			✔		✔		✔
12	Number Magic 3	✔	✔	✔			✔		✔		✔
14	Ladder Logic	✔	✔	✔	✔	✔	✔				✔
15	Dinner Table Logic	✔	✔	✔		✔	✔				✔
16	Fun with Logic 1	✔	✔	✔		✔	✔				✔
17	Fun with Logic 2	✔	✔	✔		✔	✔				✔
18	Fun with Logic 3	✔	✔	✔		✔	✔				✔
19	Splatland 1	✔	✔	✔	✔		✔		✔	✔	✔
20	Splatland 2	✔	✔	✔	✔		✔		✔	✔	✔
21	Venn Logic 1	✔	✔	✔		✔	✔	✔	✔	✔	✔
22	Venn Logic 2	✔	✔	✔		✔	✔	✔	✔	✔	✔
23	Venn Logic 3	✔	✔	✔		✔	✔	✔	✔	✔	✔
26	Hit the Target 1	✔	✔			✔		✔	✔	✔	✔
27	Hit the Target 2	✔	✔			✔		✔	✔	✔	✔
28	Hit the Target 3	✔	✔			✔		✔	✔	✔	✔
29	The 5 by 5 Array	✔	✔			✔	✔	✔			✔
30	Marbles	✔	✔		✔	✔	✔		✔	✔	✔
31	Triangular Sums	✔	✔		✔	✔		✔	✔	✔	✔

(continued)

Matrix of Critical-Thinking Skills *(continued)*

Page No.	Activity	Problem Solving	Mathematical Reasoning	Mathematical Communication	Generalization	Visual Thinking	Logic	Patterns & Sequences	Analytic Computation	Analysis	Open-Ended Problems
32	A Moving Experience 1	✔	✔		✔	✔	✔	✔		✔	✔
33	A Moving Experience 2	✔	✔		✔	✔	✔	✔		✔	✔
34	Card Tricks 1	✔	✔			✔		✔	✔	✔	✔
35	Card Tricks 2	✔	✔			✔		✔	✔	✔	✔
36	Card Tricks 3	✔	✔			✔		✔	✔	✔	✔
38	Sequences	✔	✔		✔	✔	✔	✔	✔		
39	What Comes Next? 1	✔	✔		✔	✔	✔	✔		✔	
40	What Comes Next? 2	✔	✔		✔	✔	✔	✔		✔	
41	Number Patterns 1	✔	✔		✔			✔	✔	✔	
42	Number Patterns 2	✔	✔		✔			✔	✔	✔	
43	Crack the Code 1	✔	✔		✔	✔	✔	✔	✔	✔	✔
44	Crack the Code 2	✔	✔		✔	✔	✔	✔	✔	✔	✔
45	Crack the Code 3	✔	✔		✔	✔	✔	✔	✔	✔	✔
46	Calendar Math	✔	✔			✔	✔	✔	✔	✔	✔
47	Addition Table Patterns	✔	✔		✔	✔	✔	✔	✔	✔	✔
48	Groups of Numbers 1	✔	✔				✔	✔	✔	✔	
49	Groups of Numbers 2	✔	✔				✔	✔	✔	✔	
53	How Many Triangles?	✔	✔			✔		✔	✔	✔	✔
54	The Checkerboard	✔	✔	✔	✔	✔		✔	✔	✔	✔
55	How Many Triangles in the Pentagon?	✔	✔		✔	✔		✔	✔	✔	✔
56	How Many Lines in This Hexagon?	✔	✔		✔	✔		✔	✔	✔	✔
57	The Puzzling Cube 1	✔	✔			✔		✔		✔	✔
58	The Puzzling Cube 2	✔	✔			✔		✔		✔	✔
59	Paper Folds 1	✔	✔			✔		✔		✔	✔
60	Paper Folds 2	✔	✔			✔		✔		✔	✔
61	Patterns in Geometric Numbers 1	✔	✔		✔	✔	✔	✔	✔	✔	✔
62	Patterns in Geometric Numbers 2	✔	✔		✔	✔	✔	✔	✔	✔	✔
63	Tangram Puzzler 1	✔	✔	✔	✔	✔		✔	✔	✔	✔
64	Tangram Puzzler 2	✔	✔	✔	✔	✔		✔	✔	✔	✔

Part 1: Critical Thinking and Algebra

As students advance in school, they are asked to move from working only with the concrete concepts associated with arithmetic to understanding and using the more abstract concepts of algebra. Students should be given the opportunity to use manipulatives, solve interesting problems with real-world applications, and apply reasoning and problem-solving skills to algebra-type problems.

To start off, "He Said—She Said" problems make functions and the use of variables interesting and fun for students. They should be encouraged to develop their own versions of this game and share them with the class.

"What Comes Next?" activities will help students develop critical-thinking skills and an understanding of the value and power of algebra by observing a variety of interesting visual patterns. These problems help students move from a situation where counting might be an appropriate strategy to a point where forming a generalized rule for the pattern is necessary. By moving from the concrete to a generalization, students develop mathematical thinking and reasoning skills.

Guess and check as a problem-solving strategy is appropriate at times, but it can be very frustrating. The types of problems modeled by "The Ladybug Concert" and "The Peanut Problem" encourage students to experiment with and apply different ways of looking for solutions. At the same time, they are expanding their understanding of algebra. Students should be encouraged to develop similar problems and present them to the class.

The concept of variables is very difficult for many students. The type of problem presented in the several pages of "Number Magic" fascinates students and provokes the question, "How did you know that?" *Is it magic or algebra?*

All of the activities in this chapter encourage students to practice previously learned problem-solving strategies while stretching their minds and asking them to look at alternatives to more comfortable approaches. As their mathematics and reasoning skills grow, students develop self-assurance and become more confident problem solvers.

Name _____ Date _____

He Said—She Said 1

Joe

Maria

Joe and Maria were playing the game, *He Said—She Said.* Joe calls out a number, and Maria tells him what the number turns into. Then it becomes Joe's job to figure out what rule Maria has used to get her answer. For example, Joe says **1** and Maria says **4**; Joe says **3** and Maria says **6**; Joe says **9** and Maria says **12**. Joe knows the answer: He says, "The rule is to add 3." Use the clues below to help you find a new rule.

Joe Says	Maria Says
2	**5**
6	**13**
4	**9**
0	**1**
What is Maria's rule?	

Now make up your own rule: _____

Name _____ Date _____

He Said—She Said 2

Joe

Maria

Joe and Maria were playing the game, *He Said—She Said.* Joe calls out a number, and Maria tells him what the number turns into. Then it becomes Joe's job to figure out what rule Maria has used to get her answer. For example, Joe says **1** and Maria says **4**; Joe says **3** and Maria says **6**; Joe says **9** and Maria says **12**. Joe knows the answer: He says, "The rule is to add 3." Use the clues below to help you find a new rule.

Joe Says	Maria Says
0	**–4**
6	**8**
4	**4**
1	**–2**
What is Maria's rule?	

Now make up your own rule: _____

Part 3: Number Theory and Problem Solving

The NCTM standards list "Mathematics as Problem Solving" as the first of the standards at all grade levels. Seeing mathematics as more than a collection of rules and algorithms is essential if students are to understand the power and usefulness of math. Students need experiences that (1) give them practice applying varied strategies to solve problems, (2) provide a diversity of problems, and (3) allow them to develop into confident problem solvers. The problems presented in this section offer students a variety of problems and allow for creativity in solution.

The first two "Hit the Target" activities encourage students to practice computation skills while also developing problem-solving strategies. Students may find that making a table will help in the solution of "Hit the Target 3," a strategy useful with other similar problems.

The next three problems are open-ended, and students should be encouraged to find as many solutions to each as they can. In "The 5 by 5 Array," the various solutions are, for the most part, transformations of the original. Students can be introduced to reflections and rotations at this time. "Marbles" is limited only by the number of moves students are allowed to use. Encourage students to share their strategies and solutions with the class. "Triangular Sums" is a problem with many solutions, each one a rotational transformation of the others. Again, encourage students to find as many solutions as they can.

"A Moving Experience" activities are variations of an *oldie but goodie* in the annals of problem solving. Students are given the minimum number of moves necessary to accomplish each task and should be encouraged to meet the challenge.

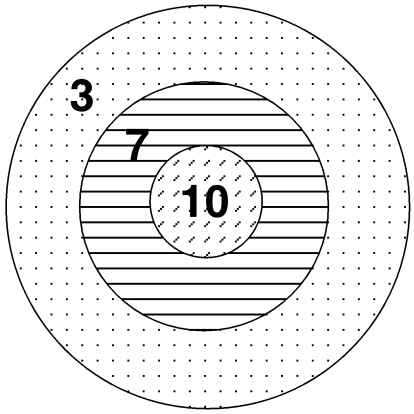
The NCTM standards recommend on page 95 that the teaching of computation skills (1) foster a solid understanding of and proficiency with simple calculations and (2) give students experiences with problems that will develop estimation techniques. The four "Card Tricks" exercises motivate students to work on open-ended problems that will help strengthen their computation and number-theory skills. As these problems have many different solutions, students should be given the opportunity to share theirs with the class. In this way the myth that there is *one right way* to solve a problem can be dispelled.

Each of the warm-ups in this section can be expanded upon, and additional puzzles can be developed by the teacher or students for use at another time.

(continued)

Name _____ Date _____

Hit the Target 1

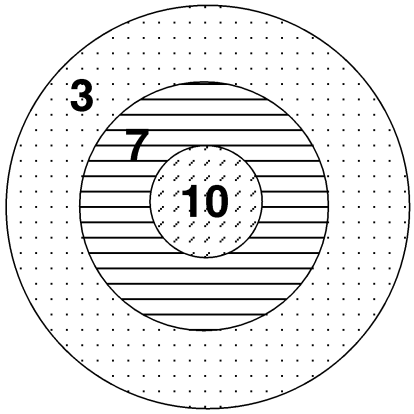


You have six shots at the target above. Show how you can achieve these scores:

- ⇨ 43 _____
- ⇨ 40 _____
- ⇨ 33 _____
- ⇨ 29 _____

Name _____ Date _____

Hit the Target 2



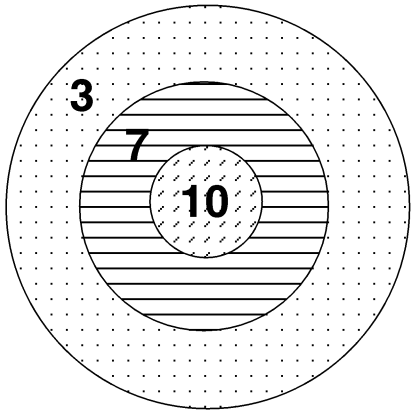
Charlie had six shots at the target above. Which of the following scores could Charlie **not have gotten?**

50 **51** **52** **53**

Explain your choice.

Name _____ Date _____

Hit the Target 3



You have six shots at the target shown above. How many different scores are possible if you hit the 10 at least once? _____

How many different scores are possible if you miss the 10 altogether? _____

(*Hint:* Use a table to organize the data.)

Name _____ Date _____

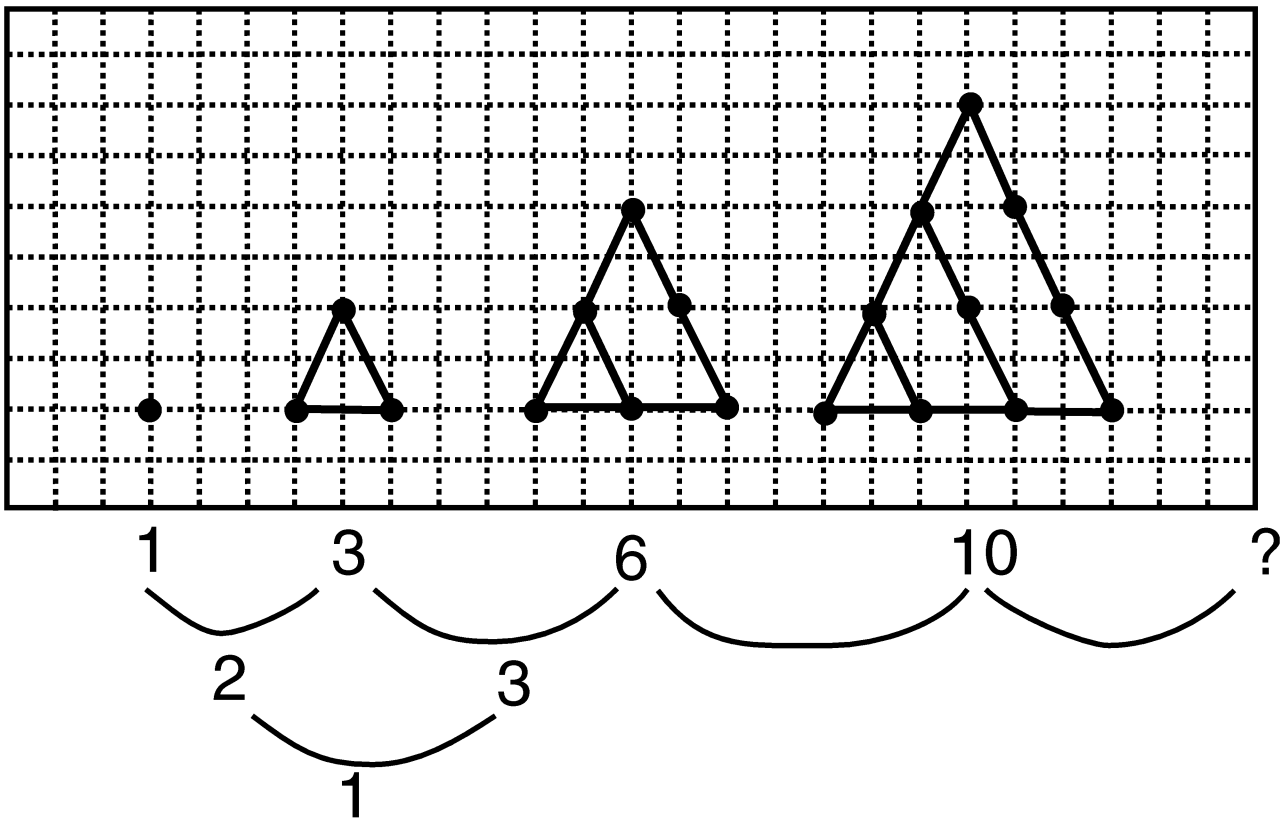
Patterns in Geometric Numbers 1

The numbers shown below are called triangular numbers. Four of them have been sketched for you. Copy the triangles shown. Then do the following:

- Sketch the next one in the series.
- Count the number of dots in each triangular figure.
- The differences in their numbers have been figured for those shown.
- The second difference has been shown for the first two figures; figure the second differences for the next two.

Can you use these number patterns to discover the number of dots in the fifth? . . . the tenth? . . . the *n*th?

Explain the pattern you discovered.



Name _____ Date _____

Patterns in Geometric Numbers 2

The numbers shown below are called square numbers. Four of them have been sketched for you. Copy the squares shown. Then do the following:

- Sketch the next one in the series.
- Count the number of dots in each square figure.
- The differences in their numbers have been figured for those shown.
- The second difference has been shown for the first two figures; figure the second differences for the next two.

Can you use these number patterns to discover the number of dots in the fifth? . . . the tenth? . . . the *n*th?

Explain the pattern you discovered.