J. WESTON
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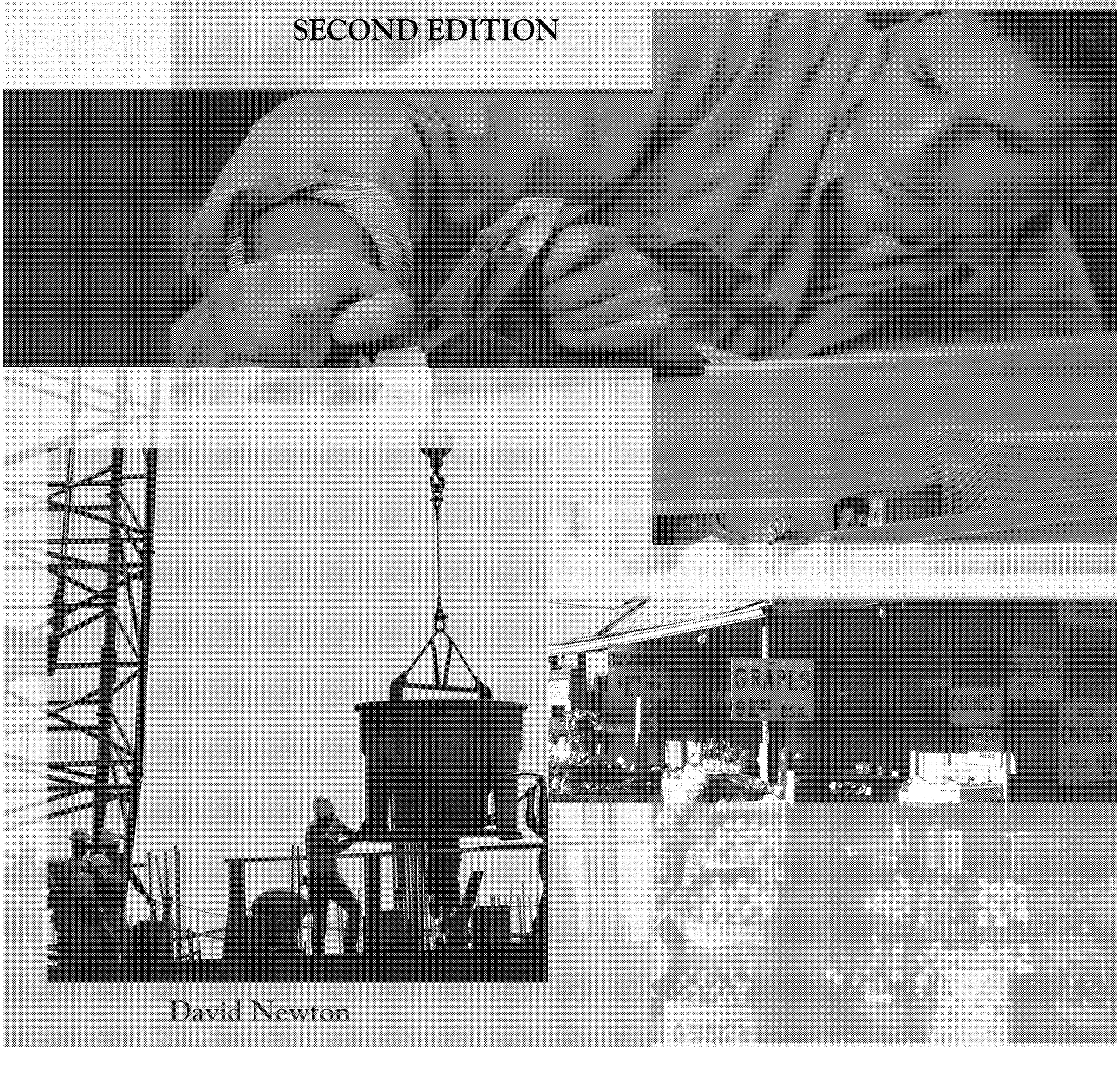
BASIC OCCUPATIONAL MATH

SECOND EDITION



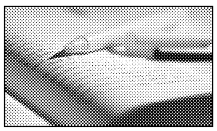
David Newton

CONTENTS

CHAPTER FOUR

POWERS, ROOTS, AND GEOMETRIC FIGURES

Roots and Powers



Learning the Concept

Any number multiplied by itself is raised to a power. For example, in the multiplication 4×4, the number 4 appears twice. So 4 is raised to the second **power.** The multiplication 4×4 can also be written another way, 4^2. In this case, the small number ^{2}, the **exponent,** tells how many times the large number 4, the **base,** appears in the multiplication.

base ⟶ 4^2 ⟵ exponent

The number 6^3 means that the base 6 is raised to the exponent, or power, 3.

$$6^3 = 6 \times 6 \times 6$$

A number raised to the second power is said to be **squared.** You can read the number 10^2 as "10 to the second power" or as "10 squared." A number raised to the third power is said to be **cubed.** You can read the number 5^3 as "5 to the third power" or as "5 cubed." There are no special terms for exponents greater than 3.

Raising a number to a power uses the multiplication skills you have already mastered. For example, suppose you are asked to evaluate 3^4. You now know that 3^4 means $3 \times 3 \times 3 \times 3$. But that is simple multiplication for you: $3 \times 3 \times 3 \times 3 = 81$. So $3^4 = 81$.

Working the other way around is more difficult. We might ask, for example, what number multiplied by itself will give 25. We can ask the same question another way: What is the **square root** of 25? This is the sign for square root: $\sqrt{}$.

We can now ask the question in the following form: $\sqrt{25}$.

Only one number multiplied by itself yields the answer 25. That number is 5, or $\sqrt{25} = 5$.

You might also be asked to find a **cube root.** That means, find the number that multiplied by itself and then again by itself (number × number × number) yields a certain value. For example, what is the cube root of 8?

Notice that you put a small 3 in the $\sqrt{}$ part of the operation symbol to write the cube root sign. Cube roots are much harder to guess than square roots. Because $2 \times 2 \times 2 = 8$, the cube root of 8 is 2.

Most calculators have a key for finding square roots and cube roots.

— Solved Examples —

EXAMPLE: Use a calculator to find the square root of 47.

SOLUTION: The problem is to find $\sqrt{47}$.

1. Clear display. [C]
2. Enter the number whose square root is to be found. [4] [7]
3. Press the square root key. [√]
4. Read the answer on the display. 6.86

Practice Problems

Use a calculator to perform each of the following operations. If there are more than two decimal places in the answer, round off to the hundredths place.

1. $(11)^2$ ___________
2. $(4.2)^2$ ___________
3. $(0.45)^2$ ___________
4. $(12)^3$ ___________
5. $(2.1)^3$ ___________
6. $(1.2)^3$ ___________
7. $\sqrt{81}$ ___________
8. $\sqrt{243}$ ___________
9. $\sqrt{0.653}$ ___________
10. $\sqrt{0.00592}$ ___________

Geometric Figures

Learning the Concept

Roots and powers are common in problems involving geometric figures. Geometric figures, such as squares, rectangles, circles, cubes, spheres, and cones, are constructed with straight and curved lines. A common assignment in many occupations is to calculate the dimensions of a geometric figure.

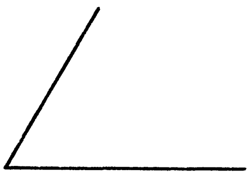
For example, a farmer may need to find the area of a field to calculate how much fertilizer to buy. Or, a contractor might have to calculate the volume of a swimming pool to meet a client's requirements.

In this section, you will learn about the most common geometric figures.

— Geometric Figures —

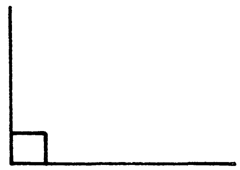
Plane figures have only two dimensions, for example, width and length.

Angle



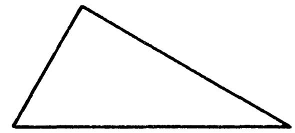
Two straight lines
that intersect (meet)
at a point.

Right Angle



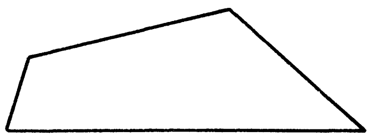
A 90° angle.

Triangle



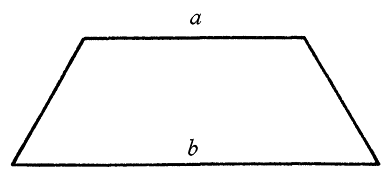
A three-sided figure.

Quadrilateral



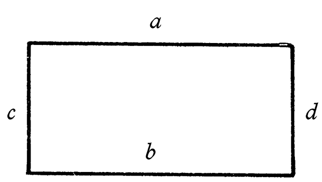
A four-sided figure.

Trapezoid



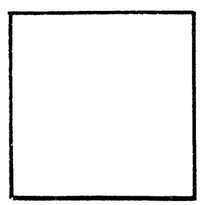
Quadrilateral with two
parallel sides (*a* + *b*).

Rectangle



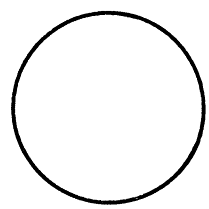
Quadrilateral with two pairs
of parallel sides
(*a* + *b*; *c* + *d*).

Square



Rectangle in which
all four sides are equal.

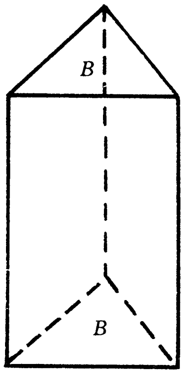
Circle



A closed curved line,
all points of which are
the same distance from
the center.

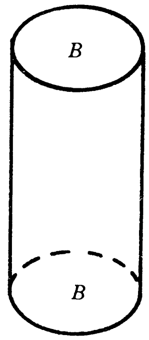
Solid figures have three dimensions, for example, length, width, and height.

Prism



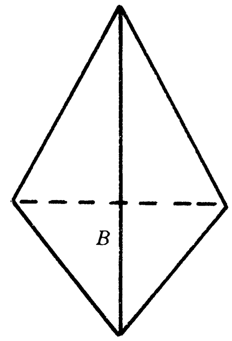
Two triangular bases (*B*).

Cylinder



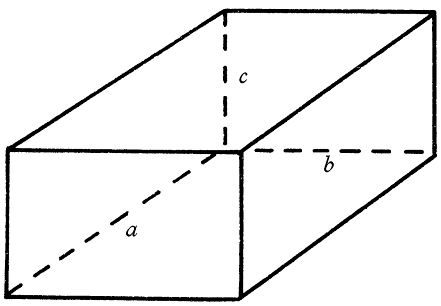
Two circular bases (*B*).

Pyramid



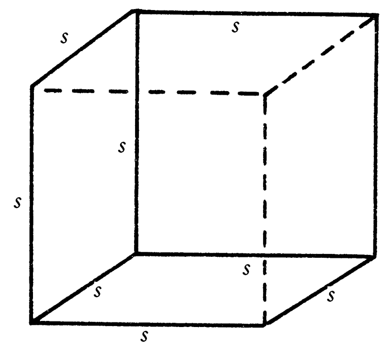
One triangular, rectangular, or square base (*B*).

Rectangular Solid



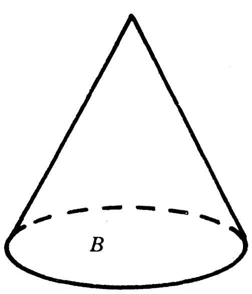
Three rectangles (*a*, *b*, *c*) at right angles to each other.

Cube



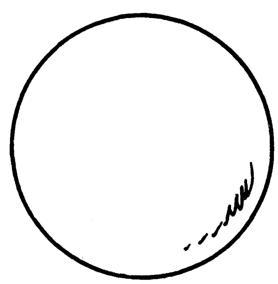
A rectangular box whose sides (*s*) are the same size.

Cone



One circular base (*B*).

Sphere



A closed surface, all points on which are the same distance from its center.

Linear, Angular and Circular Measurement

Learning the Concept

— Perimeters —

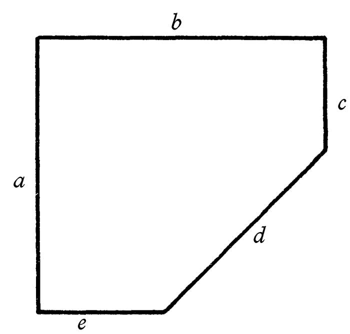


Figure 4.1

The distance around a geometric figure is called its **perimeter.** The perimeter of the cornfield shown in Figure 4.1, for example, is the sum of all five sides of the field. You can express that sum as follows:

$$p = a + b + c + d + e$$

Two widely used perimeter formulas are shown below.

Perimeter of a rectangle:	*Perimeter of a square:*
$p = 2l + 2w$	$p = 4s$
(l = length and w = width)	(s = length of one side)

Expressions such as $2l$, $2w$, and $4s$ mean $2 \times l$, $2 \times w$, and $4 \times s$, respectively. The equations below mean exactly the same thing:

$$p = 4s \qquad p = 4 \times s \qquad p = (4)(s) \qquad p = 4 \cdot s \qquad p = (4) \times (s) \qquad p = (4) \cdot (s)$$

— Angles —

Use the degree symbol [°] to express the size of an angle. A circle contains 360 degrees, or 360°. So 1° is $\frac{1}{360}$ of the way around a circle. Some common angles are shown in Figure 4.2.

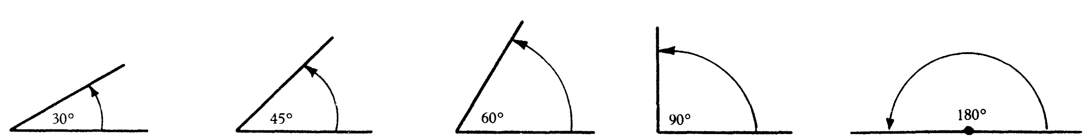


Figure 4.2

Use the minute sign ['] when you divide each degree into minutes for more precise measurements. Use the symbol for a second ["] when you divide each minute of an angle into seconds.

$$1° = 60' \qquad 1' = 60''$$

— Circumference —

The perimeter of a circle has a special name, **circumference,** usually represented by the letter *C*. The distance across a circle is called its **diameter,** indicated by the letter *D*. The distance from any point on the circumference to the center of the circle is called the **radius,** represented by the letter *r*. The diameter of any circle is equal to twice its radius, or

$$D = 2r$$

A special relationship exists between the circumference of a circle and its diameter, no matter how large or small the circle is. If you divide the circumference by the diameter ($C \div D$), you always get the same number. That number, to the nearest billionth, is 3.141592653. The number is called *pi* and is represented by the Greek letter π. Whenever you see the symbol π, just remind yourself that pi is *a number*—always the same number—and do not think of it as an unknown.

Mathematicians have now calculated the exact value of pi to many thousands of decimal places. For the problems encountered in most occupations, however, you can use the value of 3.14 for pi.

Most calculators have a key for pi, labeled π. When you use this key, instead of entering 3.14, your result will be more precise, with a value carried to many decimal places.

We express the relationship of pi to the circumference *C* and diameter *D* of a circle as follows.

$$C = \pi D \quad \text{or} \quad \frac{C}{D} = \pi$$

Because $D = 2r$,

$$C = \pi D$$

$$\text{or } C = \pi\ 2r$$

$$\text{or } C = 2\pi r$$

— Solved Examples —

EXAMPLE A: How much trim does Willard need to go around the perimeter of the floor of a room that is 14'6" long and 8'10" wide?

SOLUTION: In this case, *l*, the length of the room, is 14'6" and *w*, the width of the room, is 8'10". Use the formula for the perimeter of a rectangle, $p = 2l + 2w$.

$$p = 2l + 2w$$

$$p = (2 \times 14'6'') + (2 \times 8'10'')$$

To solve this problem, change all the measurements to feet or change them all to inches.

$$14'6'' = 14\frac{1}{2}' \text{ or } 174''$$

$$8'10'' = 8\frac{10}{12}' = 8\frac{5}{6}' \text{ or } 106''$$

Using the first of these conversions:

$$p = [(2) \times (14\tfrac{1}{2}')] + [(2) \times (8\tfrac{5}{6}')]$$

$$p = [(2) \times \left(\frac{29'}{2}\right)] + [(2) \times \left(\frac{53'}{6}\right)]$$

$$p = 29' + \frac{53}{3}'$$

$$p = 29' + 17\frac{2}{3}'$$

$$p = 46\frac{2}{3}'$$

EXAMPLE B: Use a calculator to find the circumference of a circle whose radius is 2.3 feet.

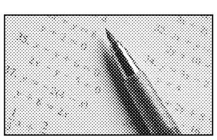
SOLUTION: First write the formula for the circumference of a circle: $C = 2\pi r$.

Then substitute the value of the radius given in the problem: $C = (2) \times (\pi) \times (2.3')$.

Then perform the indicated multiplication on the calculator.

1. Clear display. [C]
2. Enter the number 2. [2]
3. Press the multiplication key. [×]
4. Press the key marked π. [π]
5. Press the multiplication key. [×]
6. Enter the number 2.3. [2] [.] [3]
7. Press the equals key. [=]
8. Read the answer on the display. 14.45136

The circumference of the circle, rounded to the nearest tenth, is 14.5 feet.



Practice Problems

1. What is the perimeter of a triangle with sides that are 3.5", 4.2", and 6.9" long?

____

2. What is the perimeter of a rectangle with a length of 31.4 cm and a width of 17.9 cm?

____

3. How much fencing does Olga need to completely enclose her farm, shown in Figure 4.3?

____

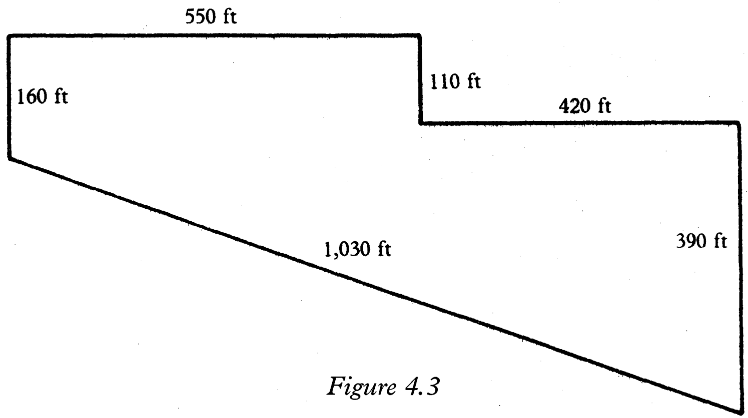


Figure 4.3

4. Amy's job at the print shop is to band together booklets in stacks of 50 each. Each booklet is 11" long, $8\frac{1}{2}$" wide, and $2\frac{3}{4}$" thick (see Figure 4.4). What length of banding material will Amy need if the bands run across the width of the stack? Across the length of the stack?

____

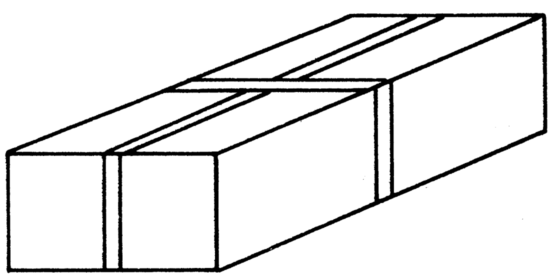


Figure 4.4

5. Velma has to paint a solid red band one quarter of the way around a silo that is 148 feet in diameter. What will the length of the band be? Round your answer to the nearest 0.1 foot.

____

6. A 1-mile racetrack is laid out in a perfect circle. Use $\pi = 3.14$ to find the track's diameter (to the nearest 0.1 foot).

____