J. WESTON WALCH PUBLISHER

top SHELF

PRECALCULUS

$$y = \log_b x$$

J. Bryan Sullivan

Contents

To the Teacher

Creative problem solving, precise reasoning, effective communication, and alertness to the reasonableness of results are some of the essential areas of mathematics that educators have specified as necessary in the development of all students to function effectively in this century. It is in the spirit of the aforementioned competencies that *Top Shelf Math* is offered to teachers of mathematics.

Top Shelf Math is intended to help students become better problem solvers. The problems presented in this format are challenging, interesting, and can easily be blended into the teaching styles and strategies of teachers who are seeking supplementary problems that support and enhance the curriculum being taught.

In general, the topics selected for each of the major content areas of *Top Shelf Math* are typical of those found in the curricula of similarly named courses offered at the high school and early college level. All of the problems presented can be used by the teacher to help his or her students improve their problem-solving skills without slowing the pace of the course in which the students are enrolled.

Other areas of utilization of the problems presented in *Top Shelf Math* could be by teachers to help prepare their students for achievement tests and advanced-placement tests. Math-team coaches will find these problems especially useful as they prepare their students for math competitions. It is recommended that students save the problems and solutions presented in *Top Shelf Math* because they provide a rich resource of mathematical skills and strategies that will be useful for preparing to take standardized tests and enrollment in future mathematics courses.

Mathematical thought along with the notion of problem solving is playing an increasingly important role in nearly all phases of human endeavor. The problems presented in *Top Shelf Math* help provide the teacher with a mechanism for the students to witness a variety of applications in a wide sphere of real-life settings.

In problems requiring a calculator solution, it is recommended that only College Board-approved calculators be used. In addition, some problems will suggest that a calculator not be used and that the solution will require an algebraic procedure.

The approach for solving the problems presented in *Top Shelf Math* is consistent with emphasis by national mathematics organizations for reform in mathematics teaching and learning, content, and application by taking advantage of today's technological tools that are available to most if not all high school and college students enrolled in similarly named courses. *Top Shelf Math* provides the teacher with a balance of using these tools as well as well-established approaches to problem solving.

We hope that you will find the problems useful as general information as well as in preparation for higher-level coursework and testing. For additional books in the *Top Shelf Math* series, visit our web site at walch.com.

INSTRUCTION

Matrices and Determinants

If two matrices have the same dimension, we can add and subtract them.

All the problems in this unit can be solved by using a graphing calculator. You should know how to solve them using a calculator, but the problems are written to solve without using a calculator.

A rectangular array of numbers expressed as $C = \begin{bmatrix} 1 & \sqrt{2} \\ 4 & \pi \\ 0 & -6 \end{bmatrix}$ is a **matrix.** Its **dimension** is determined by its number of rows and its number of columns. Matrix C has dimension $C_{3\times 2}$.

Operations:

If two matrices have the same dimension, we can add and subtract them.

Example 1

$$A = \begin{bmatrix} 3 & 4 & 1 \\ -7 & -3 & 0 \end{bmatrix},\ B = \begin{bmatrix} 4 & -8 & 12 \\ 6 & -5 & -4 \end{bmatrix}.$$

$$\therefore\ A + B = \begin{bmatrix} 7 & -4 & 13 \\ -1 & -8 & -4 \end{bmatrix} \text{ and } A - B = \begin{bmatrix} -1 & 12 & -11 \\ -13 & 2 & 4 \end{bmatrix}.$$

A matrix can be multiplied by a real number, which is called a scalar.

$$3\begin{bmatrix} 7 & -4 & 13 \\ -1 & -8 & -4 \end{bmatrix} = \begin{bmatrix} 21 & -12 & 39 \\ -3 & -24 & -12 \end{bmatrix}.$$

In order to multiply matrices, the following rule involving their dimensions must exist:

The product of a matrix $A_{a\times b}$ and matrix $B_{b\times c}$ is matrix $C_{a\times c}$. The column dimension of A must be the same as the row dimension of B, and the answer matrix C will have dimension of the row of A and the column of B.

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} u & v \\ w & x \end{bmatrix} = \begin{bmatrix} au + bw & av + bx \\ cu + dw & cv + dx \end{bmatrix}$; remember to multiply row by column.

Example 2

Multiply $\begin{bmatrix} 2 & 4 & -1 \\ 0 & -3 & 5 \end{bmatrix} \times \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$.

Call the first matrix A, the second B, and the answer C.

$\therefore\ A_{2\times 3} \times B_{3\times 1} = C_{2\times 1}$.

$$\begin{bmatrix} 2 & 4 & -1 \\ 0 & -3 & 5 \end{bmatrix} \times \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2\cdot 2 + 4\cdot 3 - 1\cdot 4 \\ 0\cdot 2 - 3\cdot 3 + 5\cdot 4 \end{bmatrix} = \begin{bmatrix} 4 + 12 - 4 \\ 0 - 9 + 20 \end{bmatrix} = \begin{bmatrix} 12 \\ 11 \end{bmatrix}.$$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is the **determinant** of matrix A and equals the number obtained from $ad - cb$.

The determinant of $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ is found by repeating the first two columns and using the above rule.

$$\begin{matrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{matrix} = aei + bfg + cdh - (gec + hfa + idb).$$

There are many applications using matrices and determinants. They are very useful in solving systems of equations.

π TRY IT Practice Activities

1. Find the value of x: $\begin{vmatrix} 2 & 3 \\ 7 & 1 \end{vmatrix} = \begin{vmatrix} 4 & 6 \\ 14 & x \end{vmatrix}$.

2. Find all values of x for which $\dfrac{\begin{vmatrix} x & 2 \\ 2 & x \end{vmatrix}}{\begin{vmatrix} x & x \\ x & 2 \end{vmatrix}} = -\dfrac{5}{3}$.

3. Solve for x and y. Express your answer as an ordered pair (x, y).
   $\begin{vmatrix} x-3 & y+1 \\ 17 & -3 \end{vmatrix} = -3$ and $\begin{vmatrix} y-2 & x+4 \\ 2 & -19 \end{vmatrix} = 18$.

4. Evaluate the determinant: $\begin{vmatrix} 0 & 2 & -3 \\ 3 & 5 & -3 \\ 1 & 2 & 0 \end{vmatrix}$.

5. Solve for x: $\begin{vmatrix} x^2 & 3x & 5 \\ 1 & 3 & 5 \\ 4 & -6 & 5 \end{vmatrix} = 0$.

6. If $A = \begin{bmatrix} 1 & 3 \\ 4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} x+1 \\ y \end{bmatrix}$, find x and y such that $A \cdot B = 3 \cdot B$.

7. Compute: $3\begin{bmatrix} 2 & -5 & 1 \\ 3 & 0 & -4 \end{bmatrix} - 2\begin{bmatrix} 1 & -2 & -3 \\ 1 & -1 & 4 \end{bmatrix}$.

8. Find the ordered pair (x, y) such that $\begin{bmatrix} x & y \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & y-4 \\ 2 & x \end{bmatrix} = \begin{bmatrix} -1 & -50 \\ 10 & 6 \end{bmatrix}$.

