J. WESTON
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topSHELF

STATISTICS

Contents

To the Teacher

Creative problem solving, precise reasoning, effective communication, and alertness to the reasonableness of results are some of the essential areas of mathematics that educators have specified as necessary in the development of all students to function effectively in this century. It is in the spirit of the aforementioned competencies that *Top Shelf Math* is offered to teachers of mathematics.

Top Shelf Math is intended to help students become better problem solvers. The problems presented in this format are challenging, interesting, and can easily be blended into the teaching styles and strategies of teachers who are seeking supplementary problems that support and enhance the curriculum being taught.

In general, the topics selected for each of the major content areas of *Top Shelf Math* are typical of those found in the curricula of similarly named courses offered at the high school and early college level. All of the problems presented can be used by the teacher to help his or her students improve their problem-solving skills without slowing the pace of the course in which the students are enrolled.

Other areas of utilization of the problems presented in *Top Shelf Math* could be by teachers to help prepare their students for achievement tests and advanced placement tests. Math-team coaches will find these problems especially useful as they prepare their students for math competitions. It is recommended that students save the problems and solutions presented in *Top Shelf Math* because they provide a rich resource of mathematical skills and strategies that will be useful for preparing to take standardized tests and enrollment in future mathematics courses.

Mathematical thought along with the notion of problem solving is playing an increasingly important role in nearly all phases of human endeavor. The problems presented in *Top Shelf Math* help provide the teacher with a mechanism for the students to witness a variety of applications in a wide sphere of real-life settings.

In problems requiring a calculator solution, it is recommended that only College Board-approved calculators be used. In addition, some problems will suggest that a calculator not be used and that the solution will require an algebraic procedure.

The approach for solving the problems presented in *Top Shelf Math* is consistent with emphasis by national mathematics organizations for reform in mathematics teaching and learning, content, and application by taking advantage of today's technological tools that are available to most if not all high school and college students enrolled in similarly named courses. *Top Shelf Math* provides the teacher with a balance of using these tools as well as well-established approaches to problem solving.

We hope that you will find the problems useful as general information as well as in preparation for higher-level coursework and testing. For additional books in the *Top Shelf Math* series, visit our web site at walch.com.

INSTRUCTION

t-Test

The population being tested must take on the characteristics of a normal distribution.

A z-test is appropriate for testing the mean of a large sample. When sample sizes are smaller, it is advisable to use a ***t*-test.** A t-test is conducted much the same as a z-test in statistical computations. In fact, t-values approach z-values as sample sizes increase. As sample sizes approach infinity, the z-value and t-value are almost identical.

In order to use a t-test, the population being tested must take on the characteristics of a normal distribution. The sample taken must be randomly chosen.

The t-value is computed using the formula below where the sample mean is $\overline{x}$, the hypothesis mean is μ_0, the standard deviation of the sample is s, and the number of elements in the sample is n.

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}$$

This value of t is compared to the area of a critical region found from a table of t-values with regard to the **degrees of freedom** and **level of significance** of the experiment.

Example

Americans drank 15 billion gallons of soft drinks in the year 2000. A magazine article claimed that the average amount consumed by each American was 84 ounces each week. Data were collected from a random sample to provide the ounces of soft drinks consumed in one week. The results are given.

74.2	53.1	102.7	31.8	45.9	55.8
29.0	93.6	100.9	81.5	61.7	99.2

Show if this data matches the claim made in the magazine using a 1% level of significance.

First, calculate the mean and standard deviation of the sample data.

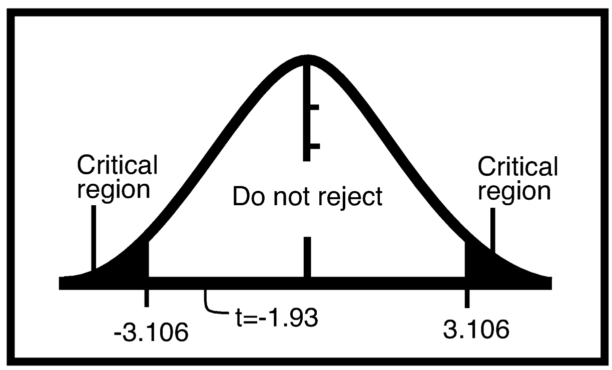
$$\overline{x} \approx 69.12 \qquad s \approx 26.72$$

The initial hypothesis H_0 is $\mu = 84$. The alternate hypothesis H_1 is $\mu \neq 84$.

A two-tailed test should be conducted with $\alpha = 0.01$. From a t-distribution table, the critical value of t with 11 degrees of freedom and $\alpha = 0.01$ is ± 3.106.

The sample t-value is calculated as shown.

$$t = \frac{69.12 - 84}{26.72/\sqrt{12}} = -1.93$$



The graph shows where the sample t-value lies in regards to the critical values of t. There is not enough information to reject the null hypothesis that the mean amount of soft drinks consumed by adults each week was 84 ounces.

π TRY IT Practice Activities

1. Use a t-distribution table to find the critical value of t for the data described.

   a. $n = 12$, right-tailed test, 1% confidence level

   b. $n = 7$, left-tailed test, 5% confidence level

   c. $n = 23$, two-tailed test, 5% confidence level

2. Studies have shown that a healthy person should receive 8 hours of sleep each night. A random sample of 14 Kellyville High School students spent one night at a sleep lab. Here are the amount of hours each student slept:

5.5	8.1	9.3	6.4	7.6	8.8	5.8
7.3	9.1	8.2	8.6	6.4	6.1	9.2

Assume that the amount of sleep all students from Kellyville get per night is normally distributed. Determine at a 5% level of significance if the population mean is less than 8 hours.

