topSHELF

TRIGONOMETRY

$c^2 = a^2 + b^2 - 2ab\cos$

$\sin\theta = \frac{opp}{hyp}$

Joseph Caruso and J. Bryan Sullivan

Contents

To the Teacher

Creative problem solving, precise reasoning, effective communication and alertness to the reasonableness of results are some of the essential areas of mathematics that educators have specified as necessary in the development of all students to function effectively in this century. It is in the spirit of the aforementioned competencies that *Top Shelf Math* is offered to teachers of mathematics.

Top Shelf Math is intended to help students become better problem solvers. The problems presented in this format are challenging, interesting, and can easily be blended into the teaching styles and strategies of teachers who are seeking supplementary problems that support and enhance the curriculum being taught.

In general, the topics selected for each of the major content areas of *Top Shelf Math* are typical of those found in the curricula of similarly named courses offered at the high school and early college level. All of the problems presented can be used by the teacher to help his or her students improve their problem-solving skills without slowing the pace of the course in which the students are enrolled.

Another area of utilization of the problems presented in *Top Shelf Math* could be by teachers to help prepare their students for achievement tests and advanced placement tests. Math-team coaches will find these problems especially useful as they prepare their students for math competitions. It is recommended that students save the problems and solutions presented in *Top Shelf Math* because they provide a rich resource of mathematical skills and strategies that will be useful for preparing to take standardized tests and enrollment in future mathematics courses.

Mathematical thought along with the notion of problem solving is playing an increasingly important role in nearly all phases of human endeavor. The problems presented in *Top Shelf Math* help provide the teacher with a mechanism for the students to witness a variety of applications in a wide sphere of real-life settings.

In problems requiring a calculator solution, it is recommended that only College Board-approved calculators be used. In addition, some problems will suggest that a calculator not be used and that the solution will require an algebraic procedure.

The approach for solving the problems presented in *Top Shelf Math* is consistent with emphasis by national mathematics organizations for reform in mathematics teaching and learning, content, and application by taking advantage of today's technological tools that are available to most if not all high school and college students enrolled in similarly named courses. *Top Shelf Math* provides the teacher with a balance of using these tools as well as well-established approaches to problem solving.

We hope that you will find the problems useful as general information as well as in preparation for higher level coursework and testing. For additional books in the *Top Shelf Math* series, visit our web site at walch.com.

INSTRUCTION

Graphical Representations

The periodic characteristics of the six trigonometric functions can be exhibited very effectively in the Cartesian coordinate system by graphing or sketching on paper.

The periodic characteristics of the six trigonometric functions can be exhibited very effectively in the Cartesian coordinate system by graphing or sketching on paper without the aid of a graphing calculator.

You should be reasonably comfortable with evaluating and finding exact values of functions at quadrantal angles and key angles such as 30°, 45°, 60°, or their radian equivalent $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$, that lie in the first quadrant. You should also be at ease with using the concept of reference angle as well as understanding that the sign of a trigonometric function depends on the quadrant in which the terminal side of the angles lies.

The problems in this section will require you to sketch graphs of trigonometric functions using the notion of vertical and horizontal translations. You will also encounter vertical and horizontal "shrinks" and "stretches." Vertical shrinks and stretches are typically identified as the amplitude of the function. Horizontal shrinks and stretches are typically identified as the period of the function. Horizontal translations are typically identified as the phase shift of the function. Vertical translations are typically identified as shifting the graph up or down.

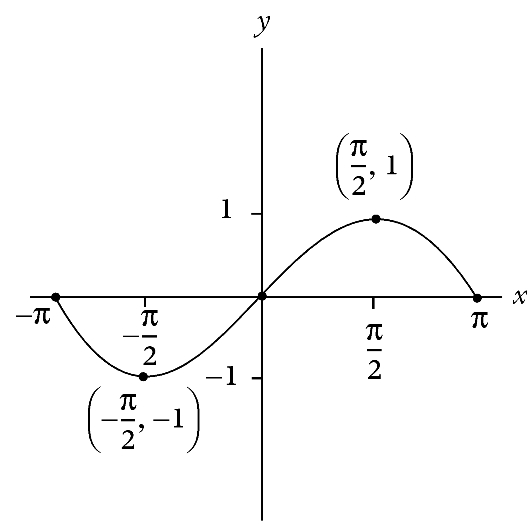
The graph of $y = f(x) \pm c$ is a vertical translation of the graph $y = f(x)$ for $c > 0$. The graph of $y = f(x) + c$ is shifted up c units, and the graph of $y = f(x) - c$ is shifted down c units. The graph of $y = f(x + c)$ is a horizontal shift of the graph of $y = f(x)$. For $c > 0$, the graph of $y = f(x - c)$ is shifted c units to the right, and the graph of $y = f(x + c)$ is shifted c units to the left.

Considering $y = a\sin(bx + c)$, a is the amplitude and its value will shrink or stretch the graph. The value $-\frac{c}{b}$ is called the phase shift, and it will cause the graph to shift horizontally. One cycle of $y = a\sin x$ is completed for $-\pi \le x \le \pi$. One cycle of $a\sin(bx + c)$ is completed by solving the inequality:

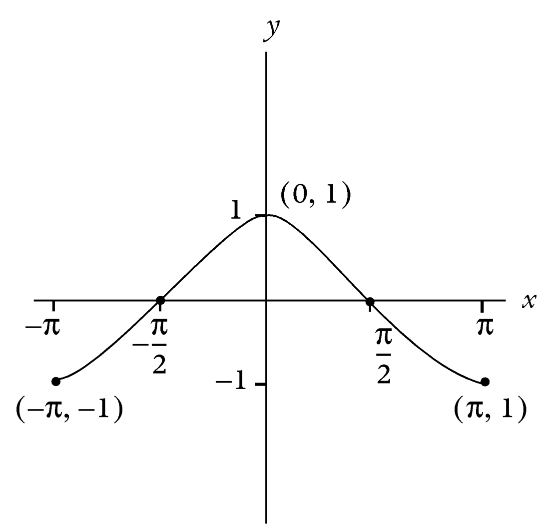
$$-\pi \le bx + c \le \pi \Rightarrow \frac{-\pi - c}{b} \le x \le \frac{\pi - c}{b}$$

Similar arguments apply to the remaining trigonometric functions. The graphs of the six trigonometric functions are sketched below. Since $\sin x = \frac{1}{\csc x}$ and $\cos x = \frac{1}{\sec x}$, the graphs of $\sin(x)$ and $\cos(x)$ are used as guides for sketching the cosecant and secant functions. It is similar for the tangent and cotangent functions. Each graph is sketched over one complete cycle of its period. Asymptotes are drawn where each function is undefined.

At the right is a sketch of $y = \sin(x)$ over the interval $-\pi \le x \le \pi$.



At the right is a sketch of $y = \cos(x)$ over the interval $-\pi \le x \le \pi$.



π TRY IT Practice Activities

For problems 1 through 12, do the following:

- Sketch one complete cycle of each given function.
- Label appropriate points on the *x*-axis and the *y*-axis.
- Label significant points that lie on the curve, such as ordered pairs that are maximum or minimum points.

1. $y = \csc\left(x - \frac{\pi}{4}\right)$

2. $y = 2\sec(2x)$

3. $y = 2\sin\left(\frac{x}{2} - \frac{\pi}{3}\right)$

4. $y = -\cos\left(2x + \frac{\pi}{8}\right)$

5. $y = 2\tan\left(\frac{x}{2} + \pi\right)$

6. $y = 3\cot(2x) + 2$

7. $y = 2\cos\left(\frac{x}{2} + \frac{\pi}{4}\right) + 1$

8. $y = -2\sin\left(3x + \frac{\pi}{4}\right) + 1$

9. $y = -\frac{1}{2}\cos\left(2x + \frac{\pi}{2}\right)$