Using a Scientific Calculator

Susan Brendel & Eugene McDevitt

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PART 5

Squares and Square Roots

Squares

The square of a number is what you get when you multiply that number by itself. For example, 5 squared is equal to 25, since $5 \times 5 = 25$. The expressions "five squared" and "5^2" mean the same thing.

♦ *The Square Key*

Look for the square key on your calculator. On some calculators you might need to press the [2nd] key to use the square key. The square key usually is labeled [x^2]. Pressing the square key squares the number in the display and leaves the result in the display. You could square a number by using regular multiplication, but using the square key will save you time and keystrokes.

Example A: Calculate 9^2, using your calculator's square key. Check your answer, using regular multiplication.

Solution: 9 [x^2] 81.

Check: 9 [×] 9 [=] 81.

Answer: 81 (Notice that using the [x^2] key saved you keystrokes.)

Example B: Calculate $(-3)^2$ using your calculator's square key. Check your answer using regular multiplication.

Solution: [(] 3 [+/–] [)] [x^2] 9.

or: 3 [+/–] [x^2] 9.

Check: 3 [+/–] [×] 3 [+/–] [=] 9.

Answer: 9

♦ *Evaluating Expressions*

Using the [x^2] key can also be helpful when you are working with longer or more complicated expressions and equations.

Example: What is the value of $6s^2 - 7$, when $s = 3$?

Solution: 6 [×] 3 [x^2] [–] 7 [=] 47.

Answer: 47

♦ *Checking Solutions of Equations*

You can use the [x^2] key when checking for solutions in algebraic equations. Often, the [x^2] key will help you to see quickly which numbers work.

Example A: Which of the following choices are solutions (roots) of $x^2 - 11 = 5$?

(a) 6, (b) 4, (c) –3, (d) –4

Solution: "Plug-in" each choice for x into the equation to see which ones are solutions.

(a) 6 [x^2] [–] 11 [=] 25. *Not a solution, since 25 ≠ 5*

(b) 4 [x^2] [–] 11 [=] 5. *Is a solution, since 5 = 5*

(c) 3 [+/–] [x^2] [–] 11 [=] –2. *Not a solution, since –2 ≠ 5*

(d) 4 [+/–] [x^2] [–] 11 [=] 5. *Is a solution, since 5 = 5*

Answer: Choices (b) 4 and (d) –4 are solutions of $x^2 - 11 = 5$.

Example B: A square with sides s feet long has an area of s^2 square feet. What is the area of the floor of a square room that has sides 12 feet long?

Solution: Use: Area = s^2, where s = 12 feet.

Substituting for s gives you: Area = 12^2 square feet

12 [x^2] 144.

Answer: 144 square feet

Try these exercises for practice with your calculator's x^2 key.

Calculate the value of each square. Check, using multiplication.

1. (a) 11^2 _____ (b) 111^2 _____ (c) 1111^2 _____

2. (a) 15^2 _____ (b) 1.5^2 _____ (c) 0.15^2 _____

3. (a) 2^2 _____ (b) 0.2^2 _____ (c) 0.02^2 _____

4. (a) $(-8)^2$ _____ (b) $(-80)^2$ _____ (c) $(-800)^2$ _____

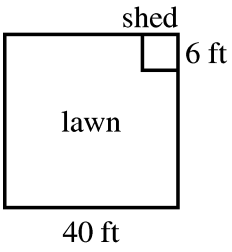
5. Evaluate each expression.
   (a) $x^2 + 7$, when $x = 9$. _____
   (b) $x^2 + y^2$, when $x = 3$ and $y = 4$. _____
   (c) $2y^2 - 17$, when $y = 3$. _____
   (d) $(x + y)^2$, when $x = 3$ and $y = 4$. _____
   (e) $3x^2 - 18x$, when $x = 7$. _____
   (f) $3x^2 - 18x$, when $x = 1{,}234$. _____

6. Which of the following choices are solutions of $x^2 - 2x = 24$? _____
   (a) 12, (b) 6, (c) –2, (d) –6, (e) –4

7. Which of the following choices are solutions (roots) of $x^2 - 2x - 8 = 0$? _____
   (a) 2, (b) 6, (c) –2, (d) –6, (e) –4

8. You are hired to cut the 40-foot by 40-foot square-shaped back lawn at a neighbor's house. There is a 6-foot by 6-foot square shed in one corner of the yard. What is the area of the lawn you have to cut?

   _____

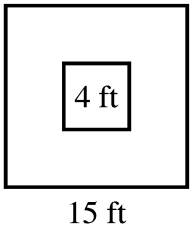


9. Nancy and Justin have been hired to tile the floor of a kitchen. The kitchen floor measures 15 feet by 15 feet.
   (a) What is the area of the floor?

   _____

   (b) There will be a 4-foot by 4-foot square island in the middle of the kitchen floor. What will be the area of the remaining floor?

   _____



Square Roots

You know how to find the square of a number. Finding a square root is like working in the opposite direction. A square root of a number is the number which, when multiplied by itself, gives you the number that you started with. For example, 5 is a square root of 25, because $5^2 = 25$ (or $5 \times 5 = 25$). The expressions "5 is a square root of 25" and "$5 = \sqrt{25}$" mean the same thing. In general:

> If $b^2 = a$, then b is a square root of a.

As with multiplying and dividing, finding a square and finding a square root are in many situations inverse operations; one seems to undo what the other one does. Calculating a square root with pencil and paper can take a lot of work. Calculating a square root with a calculator can be easy.

♦ *The Square Root Key*

Look for the square root key on your calculator. On some calculators, you might need to press the [2nd] key to use the square root key. The square root key usually is labeled [$\sqrt{x}$] or [$\sqrt{\ }$]. With most calculators, pressing the square root key calculates a square root of the number in the display and leaves the result in the display.

> When you try to find the square root of 0 or of a negative number, the calculator will display 0 or an error message.

♦ *Differences in Calculators*

There are some differences in the use of the square root key on calculators from different manufacturers. Try the following sequence of keystrokes on your calculator:

4 [$\sqrt{x}$] [×] 2 [=] _____

If the number that is in the display after pressing the [=] key is 4, then you will be able to work through the examples in this section with your calculator exactly as they are written. If there is anything other than the number 4 in the display, then look in your calculator's instruction book for information on the use of the square root key.

Example A: Calculate the value of a square root of 256.

Solution: 256 [$\sqrt{x}$] 16.

Answer: 16

Note: You might know that –16 is also a square root of 256, since $(-16)^2 = 256$. But most calculators will display only the non-negative square root of a number. In most cases, the non-negative square root is the only one that you will be interested in.

Example B: Calculate the value of a square root of 3.

Solution: 3 [$\sqrt{x}$] 1.732050808

Answer: ≈ 1.732 (to three decimal places)

Note: In Example B, 1.732050808 is not exactly equal to a square root of 3. But it is very close. It is your calculator's best approximation. When the answer is an approximation the symbol "≈" is used. That symbol means "approximately equal to," instead of "=." In most cases, a close approximation to a square root is all that you will need.

Try these problems using your calculator's $\boxed{\sqrt{x}}$ key. You do not need to include negative square roots as part of the answer. When you need to use an approximation give the answer to 3 decimal places. Use "≈" where it is needed.

1. Calculate the value of each square root. Check your answer, using multiplication or the $\boxed{x^2}$ key.

(a) $\sqrt{289}$ _____ (e) $\sqrt{64}$ _____

(b) $\sqrt{225}$ _____ (f) $\sqrt{36}$ _____

(c) $\sqrt{169}$ _____ (g) $\sqrt{16}$ _____

(d) $\sqrt{121}$ _____ (h) $\sqrt{6}$ _____

2. Calculate the value of each square root.

(a) $\sqrt{0.049}$ _____ (e) $\sqrt{490}$ _____

(b) $\sqrt{0.49}$ _____ (f) $\sqrt{4{,}900}$ _____

(c) $\sqrt{4.9}$ _____ (g) $\sqrt{49{,}000}$ _____

(d) $\sqrt{49}$ _____ (h) $\sqrt{490{,}000}$ _____

♦ *Evaluating Expressions*

Using the [$\sqrt{x}$] key can be helpful when you are working with complicated expressions and equations. Enter everything that is inside the radical symbol first. Then press the square root key.

Example A: Calculate the value of $\sqrt{5^2 + 12^2}$

Solution: 5 [x^2] [+] 12 [x^2] [=] [$\sqrt{x}$] 13.

Answer: 13

Example B: What is the value of $\sqrt{6n^2 - 5}$, when $n = 3$?

Solution: 6 [×] 3 [x^2] [−] 5 [=] [$\sqrt{x}$] 7.

Answer: 7

Try these problems, using your calculator's [$\sqrt{x}$] key. When you need to use an approximation, give the answer to 3 decimal places. Use "≈" where it is needed.

1. Calculate the value of each square root.

(a) $\sqrt{6^2 + 8^2}$ _____

(b) $\sqrt{17^2}$ _____

(c) $\sqrt{\sqrt{16}}$ _____

(d) $\sqrt{289} + \sqrt{169}$ _____

(e) $\sqrt{3 \times 27}$ _____

(f) $\sqrt{3} \times \sqrt{5}$ _____

2. Evaluate each expression.

(a) $\sqrt{x^2 - y^2}$, when $x = 13$ and $y = 5$. _____

(b) $\sqrt{3n - 6}$, when $n = 7$. _____

(c) $\sqrt{x^2 - y^2}$, when $x = 13$ and $y = -5$. _____

(d) $\sqrt{3j^2 - 3}$, when $j = 7$. _____

(e) $\sqrt{a^2 - b^2}$, when $a = 9$ and $b = 8$. _____

(f) $\sqrt{4h^2 - 28h}$, when $h = 16$. _____

Using the Quadratic Formula

Your calculator's [$\sqrt{x}$] key can be helpful when you are trying to find the roots (solutions) of quadratic equations using the quadratic formula.

The Quadratic Formula states:

If you have an equation of the form $ax^2 + bx + c = 0$, then the roots (solutions) of the equation are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: Using the quadratic formula and your calculator, find the roots of $2x^2 + 4x - 16 = 0$, and check your answer.

Solution: $2x^2 + 4x - 16 = 0$ is of the form $ax^2 + bx + c = 0$; so use the quadratic formula, where $a = 2$, $b = 4$, and $c = -16$.

After substituting in the quadratic formula, you have

$$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-16)}}{2(2)}$$

One root is given by the following sequence of keystrokes:

4 [+/–] [+] [(] 4 [x^2] [–] 4 [×] 2 [×] 16
[+/–] [)] [$\sqrt{x}$] [=] [÷] [(] 2 [×] 2 [)]
[=] 2.

and the other root is given by:

4 [+/–] [–] [(] 4 [x^2] [–] 4 [×] 2 [×] 16
[+/–] [)] [$\sqrt{x}$] [=] [÷] [(] 2 [×] 2 [)]
[=] –4.

You must enter parentheses to group everything that is inside the radical symbol and again to group everything in the divisor. Without the parentheses, your solution will not be correct.

Check: Plug each root into the original equation: $2x^2 + 4x - 16 = 0$.

2 [×] 2 [x^2] [+] 4 [×] 2 [–] 16 [=] 0.

and: 2 [×] 4 [+/–] [x^2] [+] 4 [×] 4 [+/–] [–] 16
[=] 0.

Answer: –4 and 2

When you are beginning to work with problems that require a long sequence of keystrokes, like the one above, you might be more comfortable breaking the problem

down into smaller problems. Then you can either use your calculator's memory or write down the answers to these smaller problems. You can combine these intermediate answers to come up with the final answer. Doing this might help build your confidence in using the calculator and might make checking your calculations easier.

Use your calculator's $\boxed{x^2}$ and $\boxed{\sqrt{x}}$ keys on the following problems. When you need to use an approximation, give the answer to 3 decimal places. Use "≈" where it is needed.

1. Using the quadratic formula, find the roots of the following equations. Check your answers.

   (a) $2x^2 + 6x - 20 = 0$ _____

   (b) $3x^2 + 9x - 54 = 0$ _____

   (c) $3x^2 - 6x - 9 = 0$ _____

   (d) $x^2 + 2x - 3 = 0$ _____

   (e) $3x^2 + 2x - 4 = 0$ _____

   (f) $4x^2 + 29x + 7 = 0$ _____

   (g) $2x^2 + 5x - 3 = 0$ _____

   (h) $x^2 - x - 1 = 0$ _____

2. Now find the roots of these equations. (Be careful!) Check your answers.

   (a) $2x^2 - 189x + 4{,}429 = 0$ _____

   (b) $x^2 + 1{,}842x - 271{,}123 = 0$ _____

   (c) $x^2 - 14{,}296x - 161{,}045{,}321 = 0$ _____

The Pythagorean Theorem

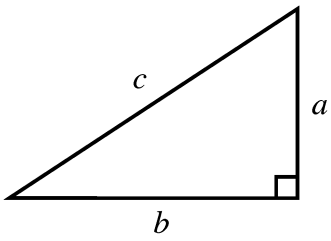
Pythagoras was an ancient Greek philosopher, astronomer, mathematician, vegetarian, and leader of a secret society. The members of the secret society, called Pythagoreans, did some very important work in mathematics. A very useful theorem, known as the Pythagorean theorem, is named after Pythagoras.

The Pythagorean theorem describes a relationship of the lengths of the sides of any right triangle. In a right triangle, the side opposite the right angle is called the hypotenuse. (The hypotenuse is the longest side of the triangle.) The other two sides are called the legs.

The Pythagorean theorem states:

In a right triangle, where a and b are the lengths of the legs of the triangle and c is the length of the hypotenuse, you have

$$a^2 + b^2 = c^2$$



Your calculator's [$\sqrt{x}$] key and [x^2] key will save you time on these right triangle problems.

Example A: A right triangle has legs of length 3 yards and 4 yards. What is the length of its hypotenuse?

Solution: Use: $a^2 + b^2 = c^2$, where $a = 3$ yards and $b = 4$ yards.

After substituting for a and b, you have $3^2 + 4^2 = c^2$.

You want to know the value of c not c^2, so take the square root of both sides.

This gives you $\sqrt{3^2 + 4^2} = c$

3 [x^2] + 4 [x^2] [=] [$\sqrt{x}$] 5.

Answer: 5 yards

Example B: Find the length of the hypotenuse of a right triangle with legs that are each 45 centimeters in length.

Solution: Use: $a^2 + b^2 = c^2$

45 [x^2] + 45 [x^2] [=] [$\sqrt{x}$] 63.63961031

Answer: ≈ 63.63961031 cm (or ≈ 63.6 cm, rounded to one decimal place)

Example C: One of the legs of a right triangle is 8 feet long; and the hypotenuse is 17 feet long. What is the length of the other leg?

Solution: Start with: $a^2 + b^2 = c^2$.

Subtract either a^2 or b^2 from both sides.

If b^2 is subtracted: $a^2 = c^2 - b^2$.

You want to know the value of a not a^2, so take the square root of both sides.

This gives you $a = \sqrt{17^2 - 8^2}$.

17 [x^2] [−] 8 [x^2] [=] [$\sqrt{x}$] 15.

Answer: 15 feet

For the following problems, when you need to use an approximation, give the answer to 3 decimal places. Use "≈" where it is needed.

1. Use your calculator to find the length of the hypotenuse (c) of each right triangle. You are given the lengths of the legs, a and b.

   (a) a = 9 inches
   b = 12 inches
   c = _____

   (d) a = 7 meters
   b = 10 meters
   c = _____

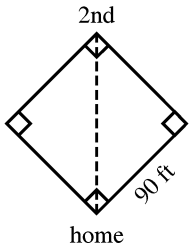
   (g) a = 3.333 feet
   b = 3.838 feet
   c = _____

2. Use your calculator to complete this table of the lengths of sides of right triangles. The first problem has been done for you. (Given legs of length 3 and 4, the length of the hypotenuse is 5.)

Legs		Hypotenuse
a	b	c
3	**4**	**5**
8	15	
6		10
	55	65
25	312	
36		85
	972	1,053
8	16	
12		36
	60,828	68,053
696	697	

For the following problems, when you need to use an approximation give the answer to 1 decimal place. Use "≈" where it is needed.

3. Assume that a baseball diamond is a perfect square, with sides that are 90 feet long. If the catcher, standing at home plate, throws the ball to the shortstop, standing at second base, how far does the ball travel from home plate to second base?



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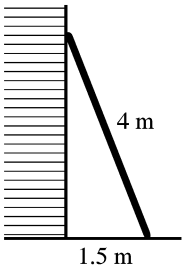
4. A rectangular playing field is 100 yards long and 55 yards wide. If Jiwon were to run diagonally across the entire field, how far would she run?

_____

5. Sandra is getting ready to paint a building. Her extension ladder is on a level sidewalk, leaning against the wall. The base of the ladder is 1.5 meters out from the base of the building. The ladder is 4 meters long. How far up the wall is the ladder touching?



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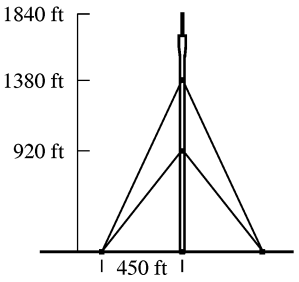
6. A radio station's transmission tower is 1,840 feet tall. Support cables are attached to the tower at a height of 920 feet and a height of 1,380 feet. The cables are anchored in the ground 450 feet from the base of the tower.



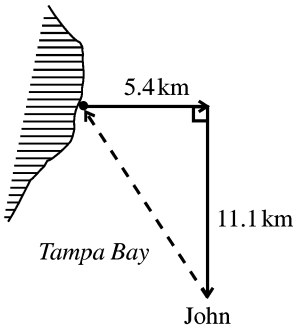
   a. How long are the cables that are attached at a height of 920 feet?

   _____

   b. How long are the cables that are attached at a height of 1,380 feet?

   _____

7. John went kayaking in Tampa Bay. He put his boat in the water and paddled 5.4 kilometers to the east. He turned and then paddled 11.1 kilometers to the south. If he wanted to travel from this point straight back to the point where he put the boat in the water, how far would he have to paddle?



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