

Hands-On Math Algebra

by

Pam Meader and Judy Storer

illustrated by
Julie Mazur

WALCH  PUBLISHING®

Contents

To the Teacher *v*

Topic: Ratio and Proportion

- 1. Candy Promotion 1
- 2. Estimating Wildlife Populations 6
- 3. The Solar System 12

Topic: Patterns and Formulas

- 4. The Picture Frame Activity 15

Topic: Integers

- 5. Integer Tiles 19
- 6. Subtraction with Integer Tiles 25
- 7. Get It in Gear Game 29

Topic: Exponents

- 8. Powers of Ten 33

Topic: Polynomials

- 9. Distributive Property 38
- 10. Discovering Algebra Tiles 43
- 11. Multiplying with Algebra Tiles 47
- 12. Factoring Trinomials with Algebra Tiles 52
- 13. Perfect Square Trinomials 59

Topic: Linear Equations

- 14. Patterns and Slope-Intercept 62
- 15. Slope as Rate 69

Topic: Graphs

- 16. Graphing Curves with a Graphing Calculator 76
- 17. How Long Is the Bounce? 80

Topic: Probability

- 18. Calendar Number Probability 84

Topic: Logarithms

19. Discovering Log Properties 86

Topic: Miscellaneous

20. Math Project: Researching Internet Providers 90

To the Teacher

Since the early 1970s we have been teaching math to learners of all ages, from young children to adults, who represent many different cultures and socioeconomic backgrounds. We believe that all learners can do math by first overcoming any math anxiety and then participating in meaningful cooperative learning activities that foster the four major standards of Communication, Problem Solving, Connections, and Reasoning. These standards are founded in the Curriculum and Evaluation Standards for School Mathematics (1989), grades K–12. The draft of the content framework for *“Equipped for the Future” Standards for Adults* (1998) suggests that for adults to be “equipped for the future,” they must be able to problem-solve, make decisions, and communicate effectively using math concepts and technology in an ever-changing world.

Our goal is to encourage all learners to “know math by doing math.” To this end, we have developed activities called “labs” that enable the learner to discover mathematical concepts through a hands-on approach. Cooperative learning skills are developed through group activities in which each learner participates collaboratively as a team member. Communication skills are fostered through group discussion and written reactions to lab discoveries. Many of the labs are connected to real-life situations. Other labs require learners to form generalizations about mathematical revelations.

As teachers, we believe learning should be learner-centered, not teacher driven. The response from our learners has been favorable. As one student said, “Thank you for turning my math disability into a math ability.”

— Pam and Judy

7. Get It in Gear Game

Learning Outcome

Students will be able to:

- perform addition and subtraction with integers.
- distinguish between an operational sign and a sign (positive or negative) for a number.

Overview

Students themselves will act as game pieces, moving back and forth on life-size number lines to perform operations with positive and negative integers.

Time

30 minutes or more

Team Size

Two students per team; two teams for each number line

Materials

Number lines taped to or drawn on the floor (one number line for every four students); cards that have operations with integers on them (see page 31)

Procedure

This activity would work best in a large area like a gym, or even outside, if feasible. This activity definitely appeals to the kinesthetic learner. The activity also clearly delineates between a sign signaling an operation (**facing**) and a sign of a number (**moving**). The activity provides clear images about *why* $-(-)$ results in $+$ and also *why* $-(+)$ and $+(-)$ have the same outcome.

1. Photocopy and cut out cards so each team has a selection to pick from.
2. Draw number lines (-10 through $+10$) with chalk or mark them with masking tape. You could have each team make their own number line first to reinforce the order of the integers on the number line.
3. Select the teams and then match up two teams for each number line; teams will take turns acting and observing.
4. Make sure the teams understand the rules and how to move on the number line. It might be beneficial to model an addition or subtraction example so that students see the difference between movement for addition and subtraction as well as direction change mandated by the signs of the numbers.

Example: The example $3 + (-5)$ would be acted out as follows: The first number is 3, so the student would stand on 3 on the number line. Addition means *face forward*, which would mean to the right or facing the front, depending on how the number line is oriented. The negative sign inside the parentheses is the *moving command*. In this example -5 means *move backward* 5 spaces. The student should land on -2 for the answer.

(continued)

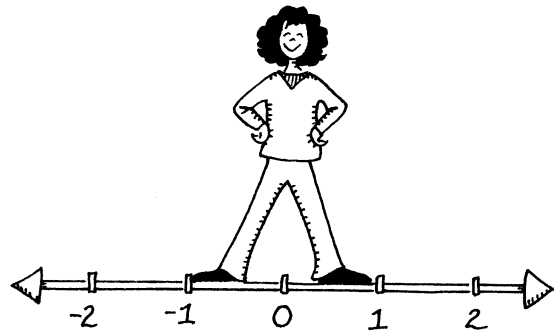
Example: $(-2) - (+3)$

The first number tells you to start at -2 on the number line. The subtraction sign means *facing the back (or left)*. The positive sign inside the next parentheses means *move forward* 3 spaces (not forward in the room, but forward in the direction you're facing). Since you are already facing in a direction, you continue in that direction. The answer will be -5 .

5. Once students are clear on how to act out each number phrase, have teams pick a card and act out the phrase on each card. At the end of each round, teams on each number line switch between acting and observing. The first team to earn 5 points wins.

7. Get It in Gear Game

Directions: Two teams will be placed by a number line on the floor. One team will act out the mathematical phrase while the other team observes and determines if the team is right. One point is awarded when a team acts out a phrase correctly. If the team acting out the phrase does it incorrectly, the observing team receives the point. The first team to receive 5 points wins the game. Each round, the teams switch roles. In one round they are the observers; the next round they act out the phrase.



How to play the game

1. Determine which team will be the observers for Round 1 and which will be acting out the phrase. In Round 2, and thereafter, the teams reverse roles.
2. Operation symbols for adding (+) and subtracting (–) are called the **facing** commands. Addition means you face **forward (right)** while subtraction means you face to the **back (left)**.
3. Symbols inside the parentheses are called the **moving** commands. The positive sign means **move forward** while the negative sign means **move to the back**.
4. The observing team draws a card from the deck and reads it to the acting team. The acting team decides how to move on the number line and finds the answer. If the answer is correct, the acting team receives 1 point. If it is not correct, the observers receive the point.
5. The first number in the phrase is where the acting team starts its player on the number line.
6. Reverse roles for Round 2, etc.
7. The first team to earn 5 points wins.

Scoreboard

	Round 1	Round 2	Round 3	Round 4	Round 5
Team 1					
Team 2					

(continued)



Activity Cards for Get It in Gear Game

$(+4) + (-3)$	$(-2) + (-3)$	$(-5) + (+4)$
$(+4) - (-3)$	$(-5) - (+3)$	$(-3) - (-3)$
$(+3) + (-3)$	$(6) + (-3)$	$(-4) + (-3)$
$(-3) + (-4)$	$(-2) - (-3)$	$(-2) - (+3)$
$(+5) + (-3)$	$0 + (-2)$	$(+2) + (-3)$
$(+6) - (-2)$	$(-2) - (0)$	$(-4) - (-3)$
$(+5) + (+3)$	$(+7) + (-3)$	$(-7) + (-3)$
$(-2) + (-3) - (+4)$	$(+5) - (-3) + (-4)$	$(-2) - (-3) - (+4)$

8. Powers of Ten

Activity One

Compute the following on your calculator.

1. $1 \times 10 =$ _____
2. $10 \times 10 =$ _____
3. $10 \times 10 \times 10 =$ _____
4. $10 \times 10 \times 10 \times 10 =$ _____
5. $10 \times 10 \times 10 \times 10 \times 10 =$ _____



Using the EXP or EE key on your calculator, find the following:

- | | |
|-------------------|--------------------|
| 6. $10^1 =$ _____ | 9. $10^4 =$ _____ |
| 7. $10^2 =$ _____ | 10. $10^5 =$ _____ |
| 8. $10^3 =$ _____ | |

What do you notice about the answers to 1–5 and the answers to 6–10?

What rule(s) can you make about powers of 10 from your observations above?

Activity Two

With your partner and a calculator, try the following examples.

- | | |
|--------------------------------|--------------------------------|
| 1. $34.567 \times 100 =$ _____ | 3. $34.5 \times 1,000 =$ _____ |
| 2. $0.234 \times 10 =$ _____ | 4. $1.2 \times 10,000 =$ _____ |

8. Powers of Ten *(continued)*

With your partner, discuss any patterns you see. Summarize them below.

Rewrite the examples on the previous page using powers of 10.

5. $34.567 \times 100 = 34.567 \times 10^2 =$ _____

6. $0.234 \times 10 = 0.234 \times 10^1 =$ _____

7. $34.5 \times 1,000 = 34.5 \times 10^3 =$ _____

8. $1.2 \times 10,000 = 1.2 \times 10^4 =$ _____

What do you notice about the exponent and the number of decimal places moved in the answer?

Use the rules that you discovered to multiply the following example without a calculator.

$$3.456 \times 10^2 = \underline{\hspace{2cm}}$$

Write a **generalization** for multiplying numbers by powers of 10 with positive exponents:

What do you think will happen when you divide by powers of 10? Write your prediction below.

Try the following examples with your calculator and see if your prediction is correct.

9. $76.589 \div 100 =$ _____

10. $9.422 \div 1,000 =$ _____

11. $0.878 \div 10^3 =$ _____

12. $45.678 \div 10^4 =$ _____

8. Powers of Ten *(continued)*

Was your prediction correct? Write a **generalization** for dividing a number by a power of 10 with positive exponents:

Activity Three

Enter the following into your calculator. Write the answers below.

1. 76.589×10^{-3} _____ 3. 12.5667×10^{-4} _____
2. 345.67×10^{-2} _____

What do you notice about the exponent and the number of decimal places moved in the answer? In which direction does the decimal point move?

Write a rule for multiplying by powers of 10 with negative exponents:

What do you think will happen if you divide by powers of 10 with negative exponents?

Try the following examples to see if you are correct.

4. $76.589 \div 10^{-3}$ _____ 6. $12.5667 \div 10^{-4}$ _____
5. $345.67 \div 10^{-2}$ _____

Write a rule for dividing by powers of 10 with negative exponents:
