Hands-On Math
Data Analysis and Probability

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Contents

To the Teacher

As teachers, we know that students learn best when they "know math by doing math." The activities in this book are designed to enable students to discover the concepts of probability and statistics through a hands-on approach. Many of the activities call for working in groups, to give students the support of others as they explore unfamiliar concepts; this also helps develop cooperative learning and communication skills.

Several major topics in probability and statistics are included in this book. These activities will lead the learner through the topic in an active, hands-on, minds-on approach to learning.

Each topic is addressed by a linked series of activities. Each part in the series builds on the activity before, but the multi-part structure makes it easy to present the series over the course of several classes. Each activity series includes a teacher page and one or more student activity pages, which contain:

- Student learning outcomes
- Time requirements
- Materials list
- NCTM Standards (2000) being addressed
- Prerequisites (if any)
- Overview and background information
- Procedure
- Reproducible student activity sheets

The activities in this book use several formats for cooperative learning. They include using a team format:

- to collaboratively construct meaning to a mathematical concept
- to generate oral and written responses for assessment
- to generate team and individual responses

This is accomplished through the intricate development of the activities. Different aspects of the team learning approach include:

- individual assignments, both written and oral

- team products
- individual assessment opportunities

Some of the collaborative learning structures include:

- The learning "pyramid"
- Jigsaws (regular and "reverse")
- Individual student responsibilities within the team

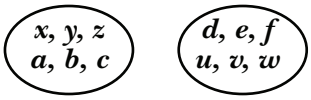
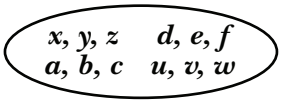
Grouping Strategies for Cooperative Learning

Throughout this book, grouping strategies are keyed to the activities. Teams designed to be homogeneous or heterogeneous have always been popular with teachers, but research indicates that this strategy has many inherent problems. Students tend to object to teams that are formed strategically. Randomly formed teams, on the other hands, not only tend to be more exciting but also nullify student complaints because the team assignments last for only a short span of time. These activities are mostly designed for such an experience. Therefore, the teams for all these activities are to be selected randomly and last for only one activity (unless stated otherwise).

You can use a deck of cards to form teams randomly. With the seating arranged according to the requirements of the activity, assign numbers to the work stations. Reduce the deck of cards so that there is a one-to-one correspondence with the specific class scheduled. Randomly hand out the cards as the students arrive, and have the students sit at the appropriate work stations according to the number on their card—aces sit at table 1, etc. By identifying the suit (e.g., "hearts"), individual jobs within the team can be assigned as the activity begins.

Some activities may require a jigsaw approach to learning. An easy method to assign responsibilities within a team (especially for a jigsaw activity) is to have students assign themselves numbers between 1 and 4. (Be sure to identify the number chosen by each student before continuing.) Students may then be assigned to tasks by common numbers. (See graphic organizer that follows.)

<table>
<tr><th colspan="4">Grouping Strategies</th></tr>
<tr><td colspan="2">Jigsaw Grouping Strategy</td><td colspan="2">Pyramid Grouping Strategy</td></tr>
<tr><td>Step 1:</td><td>Form teams.</td><td>Step 1:</td><td>x y z</td></tr>
<tr><td>Step 2:</td><td>Member w does task 1.
Member x does task 2.
Member y does task 3.
Member z does task 4.</td><td>Step 2:</td><td>xyz</td></tr>
</table>

Step 3:	Convene member *w* from each team, member *x* from each team, member *y* from each team, and member *z* from each team to share results.	Step 3:	
Step 4:	Each original team reconvenes.	Step 4:	

Constructivist Learning Theory

The structure upon which the geometry activities in this book are designed is the constructivist learning theory. The constructivist learning cycle phases—*exploration, identification,* and *application*—are frequently used. Research and classroom practice have both produced strong support for this way of learning. Students, teachers, parents, and administrators have seen positive changes in achievement, the ability to apply learning, and the reduction of anxiety.

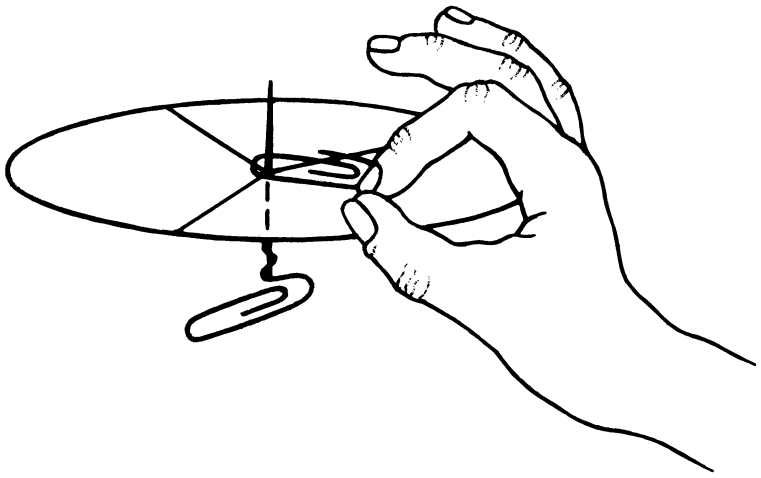
The *concept-exploration stage* gives students the chance to experience a concept with a hands-on, minds-on activity. Students begin by using their prior knowledge and innate abilities to construct their own knowledge about a concept within their own familiar framework. Reasoning skills, problem-solving aptitudes, and communication abilities are enhanced when learners can explore, experiment, and share. Many of the activities are designed for this stage of the learning cycle.

The *concept-identification stage* is the central part of the learning cycle. After exploring a concept, students engage in activities that synthesize and build on their learning. This stage helps the student develop a sophisticated level of mathematical literacy and master the abilities that will produce the learning outcome you're looking for.

The *concept-application stage* allows students to use their mathematical learning. Students *do* mathematics by applying their knowledge to solve problems, create products, or explore a new concept.

These three stages often overlap. Each activity allows for the diversity of student experiences, knowledge, and motivation. Many activities incorporate two or more stages of the learning cycle.

Spinnering



Teacher Page

Spinnering

Learning Outcomes

Students will demonstrate an ability to:

- construct a sample space from an event.
- form a simple probability ratio.

Time Requirements

90 minutes

Materials

- Spinnering handouts
- Construction paper
- Two paper clips per team
- Masking tape
- Scissors

NCTM Standards (2000) Addressed

Data Analysis and Probability:

- Select and use appropriate statistical methods to analyze data
- Develop and evaluate inferences and predictions that are based on data
- Understand and apply basic concepts of probability

Prerequisites

Students should be familiar with the concept of sample spaces.

Overview

This group of activities will lead students to understand sample spaces through experimentation. Students will make spinners, then use them to generate sample spaces. The final activity in the group will introduce forming a probability ratio.

Procedures

Part One

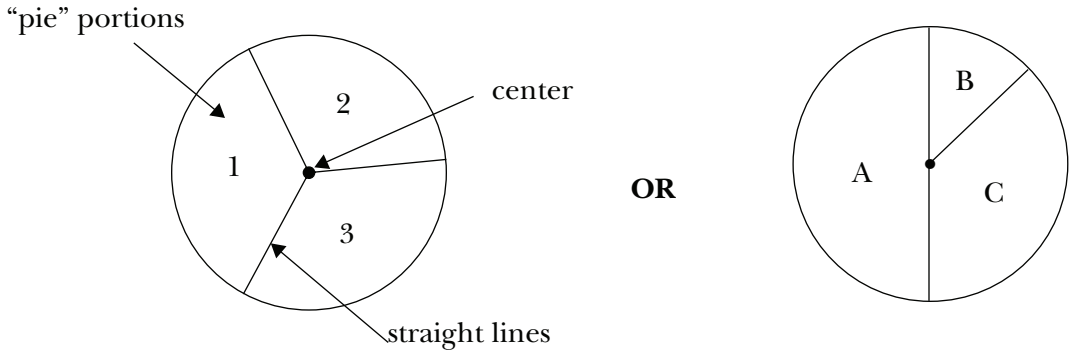
Suggested time: 15 minutes

1. Divide class into pairs.

2. Distribute Activity Sheet 1 and other materials. Review and clarify directions. Students prepare spinners according to the directions on the handout.

(continued)

3. As students complete their spinners, check that spinners have been made correctly.



4. Distribute Activity Sheet 2. Go over the directions on the activity sheet, clarifying as needed.

5. Students complete the activity sheet; this should take no more than 15 minutes.

6. As teams complete the activity sheet, assess their work. If some student pairs seem confused, use questioning to focus their responses:

   - "What could the spinners show if they were spun at the same time?"
   - "Have you listed all of the possible combinations?"

Answers to Activity Sheet 2:

FIRST SPINNER	SECOND SPINNER
1	A
1	B
1	C
2	A
2	B
2	C
3	A
3	B
3	C

(continued)

Name_____ Date _____

ACTIVITY SHEET 1

Spinnering

1. Assign each partner in your team a letter, A or B.

   Letter assigned to you:_____

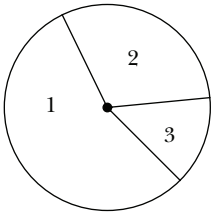
2. On the construction paper, draw a circle with a radius of 2 inches.

3. With the scissors, cut out the circle.

4. Using the center of the circle as a focus:

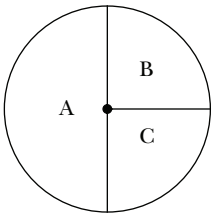
Partner A:

Draw three regions of any size within your circle. Label the regions 1, 2, and 3. Your spinner may look like this, for example:

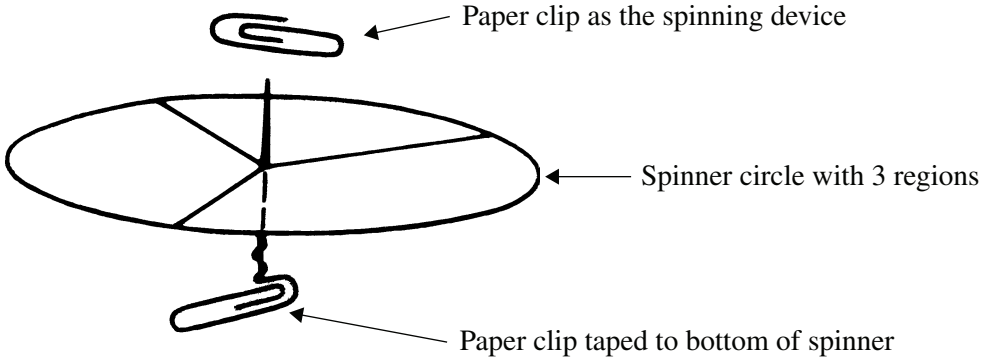


Partner B:

Draw three regions of any size. Label the regions A, B, and C. Your spinner may look like this, for example:



5. Assemble the spinners as shown below.



6. Bring the spinners to your teacher to check that they have been made correctly.

Name_____ Date _____

ACTIVITY SHEET 2

Spinnering

Draw your spinner in the space below:

Draw your partner's spinner in the space below:

Consider the spinner with the numbers as the first spinner. The spinner with the letters will be the second spinner. In the space below, list all the possible outcomes if both spinners were spun at the same time.

Possible Outcomes

FIRST SPINNER	SECOND SPINNER

Name_____ Date _____

ACTIVITY SHEET 3

Spinnering

In the space below, display your **sample space**. Your work will be assessed according to the criteria at the bottom of this page.

PRESENTATION:

0—No work.

1—Work submitted.

2—Work is somewhat neat *or* organized.

3—Work is neat *or* organized.

4—Work is neat *and* organized.

QUALITY (SAMPLE SPACE):

0—No work

1—Work submitted.

2—Some correct outcomes are listed in the sample space.

3—Most outcomes are listed correctly.

4—All outcomes are listed correctly.

Comments: _____

_____

_____

_____

Name_____ Date _____

ACTIVITY SHEET 4

Spinnering

In the first column, list the outcomes from your sample space on Activity Sheet 2.

Suppose you were to spin your spinners 50 times. In how many of those 50 times would you expect each outcome in Column 1 to occur? Write your best estimate in Column 2. Be sure that the numbers you list in Column 2 total 50.

COLUMN 1 OUTCOMES	COLUMN 2	COLUMN 3 (ACTIVITY SHEET 5)

Analysis:

_____

_____

_____

_____

_____

(continued)

Name_____ Date _____

ACTIVITY SHEET 5

Spinnering *(continued)*

1. Spin the spinners 50 times.
2. Record each outcome on the back of this sheet. For example, (1,X) is one of the outcomes.
3. In Column 3 on Activity Sheet 4, record the number of times each outcome actually occurred out of the 50 spins.
4. In the Analysis section on Activity Sheet 4, write down your thoughts about why there were differences between the numbers in Column 2 and the numbers in Column 3.

Your work will be assessed according to the following criteria:

PRESENTATION:

0—No work.

1—Work submitted.

2—Work is somewhat neat *or* somewhat organized.

3—Work is neat *or* organized.

4—Work is neat *and* organized.

QUALITY:

0— No work.

1— Work submitted.

2— Work displays some understanding of the concepts of chance.

3— Work displays an understanding of the concepts of chance, greater chances of occurrence, and the connection between spatial figures and expected outcomes.

4— Work displays achievement beyond expectations.

GRADING TRANSFER

POINT AVERAGE PER OUTCOME:

From (not including) to	Letter grade	From (not including) to	Letter grade
3.1–4.0	A	1.9–2.4	C
3.0–3.1	A–	1.8–1.9	C–
2.9–3.0	B+	1.7–1.8	D+
2.6–2.9	B	1.2–1.7	D
2.5–2.6	B–	1.1–1.2	D–
2.4–2.5	C+	1.1 or below	F

Name_____ Date _____

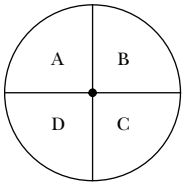
ACTIVITY SHEET 6

Spinnering: An Introduction to Simple Probability

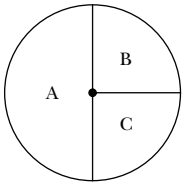
Probability *(p)* is a measure of how likely an event is. The probability of an event is a number between 0 and 1. Mathematically, it is the ratio of the number of ways an event can happen (m) to the number of possible outcomes (n). This ratio is written as $p = m/n$.

For example, look at Spinner 1 below. Suppose that you spin the pointer. What is the likelihood that the pointer will stop on A? The probability is $\frac{1}{4}$. The pointer is likely to stop on A one time out of every 4 spins.

Now look at Spinner 2 below. What is the probability that the pointer will stop on A? It is $\frac{1}{2}$. For every 2 spins, it is likely that the pointer will land on A one time.



Spinner 1



Spinner 2

Find the probabilities *(p)* for each of the following:

Spinner 1

1. p (B) = _____
2. p (C) = _____
3. p (D) = _____
4. p (Consonant) = _____

Spinner 2

5. p (B) = _____
6. p (C) = _____
7. p (Consonant) = _____

8. Draw a spinner with three sections labeled A, B, and C. The outcomes for A, B, and C should produce these probabilities:

   p (A) = $\frac{1}{3}$ p (B) = $\frac{1}{2}$

   Use the back of this paper for your drawing.

9. What is the probability of the pointer landing on C?

   p (C) = _____