# Quadratics

## Set 1: Quadratic Transformations in Vertex Form

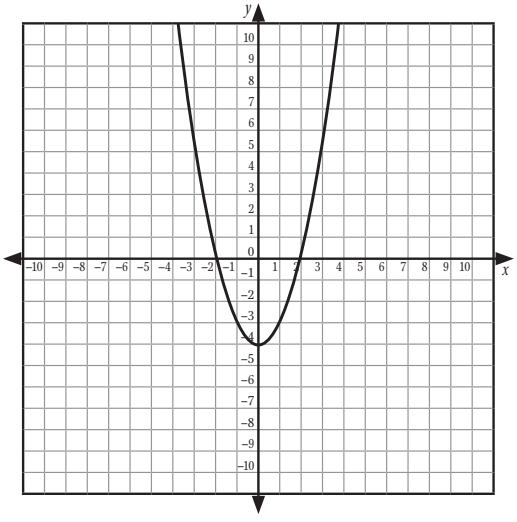
6. The equation for this parabola is $y = \frac{x^2}{2} - 4x + 9$. Find the vertex in order to convert the equation to vertex form. Show your work.

7. What are the coordinates of the *y*-intercept?

8. What is the equation for the axis of symmetry?

9. Does a parabola's axis of symmetry always run through its vertex? Why or why not?

10. Look at the graph below, which shows the parabola $y = x^2 - 4$. The coordinates of the parabola's *x*-intercepts are (2, 0) and (–2, 0). How could you use this information to find the coordinates of the parabola's vertex? Explain, showing your work.



# Quadratics

## Set 1: Quadratic Transformations in Vertex Form

### Station 4

Work with your group to answer the following questions.

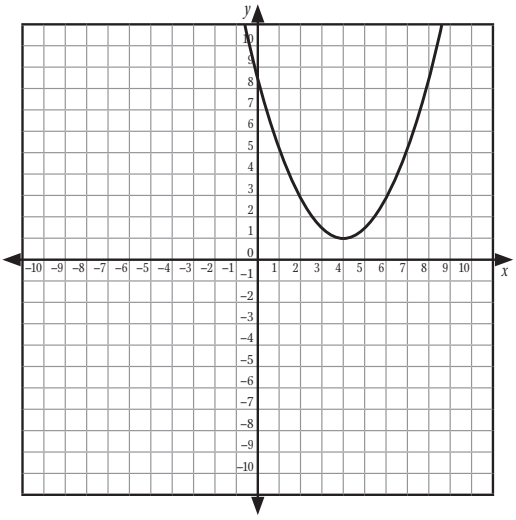
1. Graph the parabola $y = x^2 + 6x + 7$ on graph paper.

2. Give the equation for its axis of symmetry.

3. *Optional:* Complete the square to give the equation for the parabola in vertex form. Show your work.

4. What are the coordinates of the vertex of this parabola?

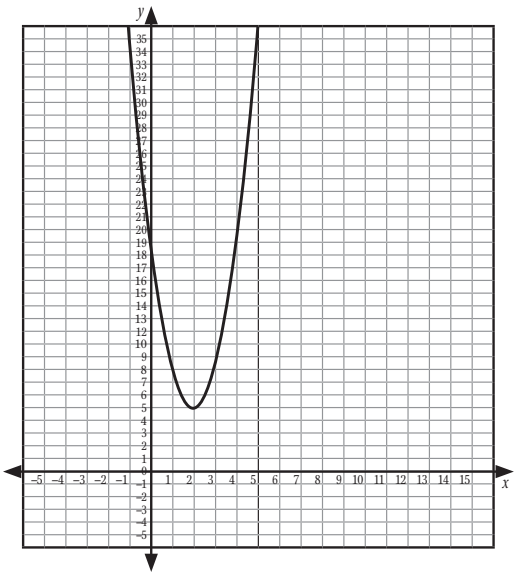
5. Look at the graph below. What are the coordinates of the vertex of this parabola?




*continued*

6. Do you think that the graph of $y = \frac{1}{2}(x-2)^2$ will be wider or narrower than the graph of $y = (x-2)^2$? Why? Graph both parabolas, in contrasting colors, to check your answer.

7. Look at the graph below. The equation for this parabola is $y = 3(x-2)^2 + 5$. What is its $y$-intercept?




8. How would you write the equation for a similar parabola with a $y$-intercept 5 units higher? Show your work. Write out an explanation in words if necessary.

# Quadratics

## Set 1: Quadratic Transformations in Vertex Form

### Station 3

Work with your group to answer the following questions.

1. Complete the table for the parabola $y = 2(x - 1)^2 + 3$. Graph the parabola on graph paper.

| $x$ | $y$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| –1 | |
| –2 | |
| –3 | |

2. What is the equation for this parabola's axis of symmetry?

3. What are the coordinates of this parabola's $y$-intercept?

4. How would this graph change if the parabola's equation changed to $y = -2(x - 1)^2 + 3$? Graph the new parabola to check your answer.

5. What are the coordinates of the $y$-intercept of the parabola $y = \frac{1}{2}(x - 2)^2 + 1$?

*continued*

6. On the same graph, in a contrasting color, graph the parabola $y = -2x^2 + 2$. Label each parabola.

7. What are the coordinates of the $y$-intercept of $y = -2x^2 + 2$?

8. What happens to the graph of a parabola when you add a numeric constant to its equation, as in problem 6?

9. What happens to the graph of a parabola when the $x^2$ expression is given a numeric coefficient, as in problems 1 and 5? (*Hint:* Compare the parabola $y = 3x^2$ to $y = x^2$.)

# Quadratics

## Set 1: Quadratic Transformations in Vertex Form

### Station 2

Work with your group to explore the relationship between a quadratic function and its graph.

1. Given the equation $y = 3x^2$, complete the table with the values of $y$ and graph the parabola.

| $x$ | $y$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| –1 | |
| –2 | |
| –3 | |

2. What are the coordinates of this parabola's $y$-intercept?

3. What is the equation of its axis of symmetry?

4. On the graph from problem 1, draw the parabola $y = x^2$ in a contrasting color. In words, compare the two parabolas.

5. Graph the parabola $y = -2x^2$. Complete the table if you need a reference.

| $x$ | $y$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| –1 | |
| –2 | |
| –3 | |

*continued*

5. What is the equation for this parabola?

6. How would you write the equation for a similar parabola of $y$-intercept $(0, -5)$?

7. Graph the parabola from problem 6.

8. Graph the parabola $y = (x - 2)^2$.

9. What is the equation for the axis of symmetry of this parabola?

10. Without graphing, predict the equation for the axis of symmetry of the parabola $y = (x + 3)^2$.

# Quadratics

## Set 1: Quadratic Transformations in Vertex Form

### Station 1

Work as a group to answer the questions. Construct graphs without the aid of a graphing calculator. Show all your work and label the axes of each graph.

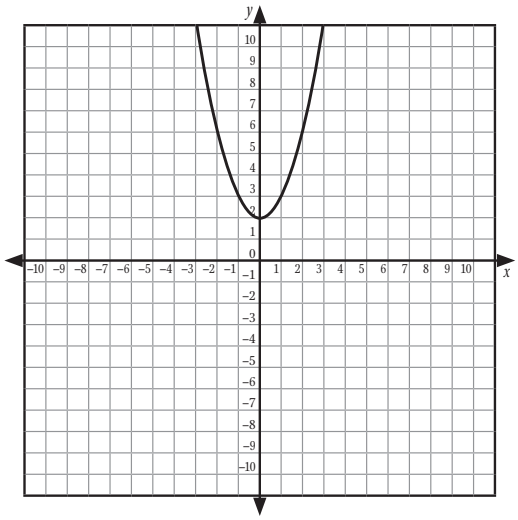
1. Given the parabola $y = x^2$, complete the table below with the $y$ coordinates for the following values of $x$.

| $x$ | $y$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| –1 | |
| –2 | |
| –3 | |

2. Use the coordinates from your table to graph the parabola on graph paper.

3. What are the coordinates for the parabola's $y$-intercept?

4. Look at the parabola below. What is its $y$-intercept?




*continued*

## Instruction

- Making simple arithmetical errors in completing the square
- Not understanding the arithmetical manipulations involved in completing the square
- Confusing the $y$-intercept with the $x$-intercept
- Confusing the vertex coordinates $h$ and $k$
- English language learners may struggle with the questions that ask for written explanations. Encourage these students to write out the numeric operations involved and then describe their work out loud.

Instruction

## Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

### Prompts/Questions

1. What is a function's axis of symmetry? Does every parabola have one?
2. What is a *y*-intercept?
3. How do you find the coordinates of a parabola's *y*-intercept?
4. Compare the equations $y = a(x - h)^2 + k$ and $y = ax^2 - 2axh + ah^2 + k$. Do you think they express the same thing? How could you find out?

### Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

### Suggested Appropriate Responses

1. An axis of symmetry is the line that divides the graph of the function into two symmetrical halves. Every parabola has an axis of symmetry.
2. A *y*-intercept is the point at which a function crosses the *y*-axis.
3. Set *x* equal to 0 and solve the function for *y*.
4. Students should multiply out the equation in vertex form to find the equation in quadratic form.

### Possible Misunderstandings/Mistakes

- Incorrectly calculating the value of *y*-coordinates from *x*-coordinates
- Incorrectly graphing parabolas, either from incorrect calculations or from a misunderstanding of graphing itself
- Not understanding the definition of the vertex
- Assuming that the vertex is unrelated to the axis of symmetry
- Incorrectly factoring quadratic equations

3. $y = x^2 + 6x + 7$

   $y = x^2 + 6x + 7 + 2 - 2$

   $y = (x^2 + 6x + 9) - 2$

   $y = (x + 3)^2 - 2$

4. (–3, –2)
5. (4, 1)
6. $y = \frac{x^2}{2} - 4x + 9$

   $y = \frac{x^2}{2} - 4x + 8 + 1$

   $y = \frac{1}{2}(x^2 - 8x + 16) + 1$

   $y = \frac{1}{2}(x - 4)^2 + 1$

7. (0, 9)
8. $x = 4$
9. Yes. The parabola opens out from the vertex. The vertex contains the only *y*-coordinate that is not repeated in the range.
10. Because the parabola is symmetrical, the axis of symmetry will intersect the midpoint of the line between the *x*-intercepts. The midpoint is at (0, 0). That means the *x*-coordinate at the vertex must be 0, because the axis of symmetry intersects the vertex. If $x = 0$, $y = -4$, so the coordinates of the vertex are (0, –4).

## Materials List/Setup

**Station 1** student activity sheet; graph paper

**Station 2** student activity sheet; colored pencils or pens; graph paper

**Station 3** student activity sheet; colored pencils or pens; graph paper

**Station 4** student activity sheet; graph paper

7. (0, 17)
8. $y = 3(x - 2)^2 + 10$

   The new $y$-intercept is (0, 22), so solve for the value of $a$.

   $y = 3(x - 2)^2 + a$

   $22 = 3(0 - 2)^2 + a$

   $22 = 3(4) + a$

   $22 = 12 + a$

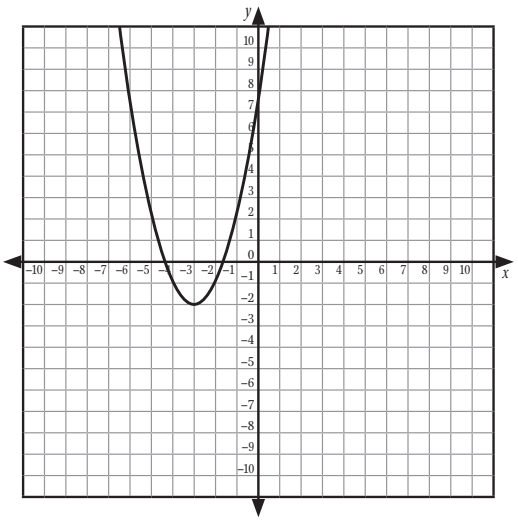
   $10 = a$

### Station 4

Students use two methods (completing the square and finding the midpoint of the $x$-intercepts) to convert the equations of parabolas from quadratic form to vertex form. They graph to check their work and to understand the correlation between the different forms and the graph. Students should recognize that a parabola's axis of symmetry always runs through its vertex. They should also understand the relationship between the coordinates of the vertex and the equation in vertex form.

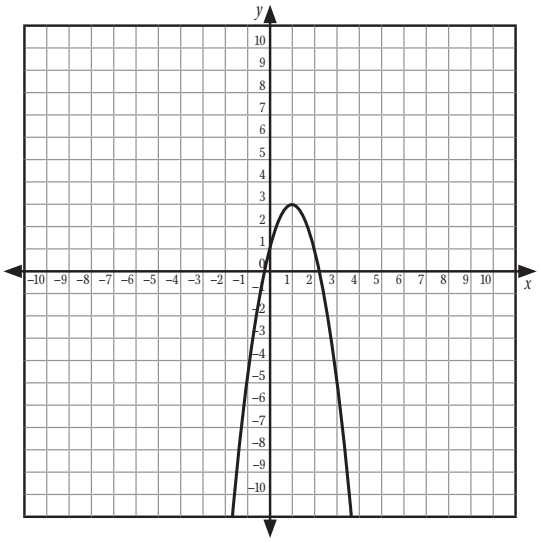
### Answers

1.




2. $x = -3$

4. The parabola would open downward.




5. (0, 3)

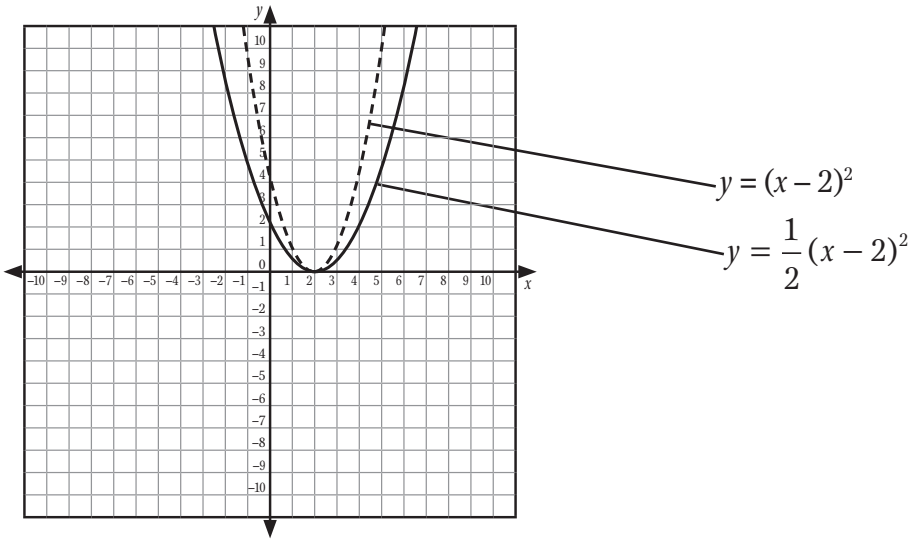
$$y = \frac{1}{2}(x-2)^2 + 1$$

$$y = \frac{1}{2}(0-2)^2 + 1$$

$$y = \frac{1}{2}(4) + 1$$

$$y = 3$$

6. It will be wider, because the higher the coefficient of the $x^2$ expression, the narrower the parabola. The coefficient of the second $x^2$ expression is 1, which is higher than ½, the coefficient of the first $x^2$ expression.



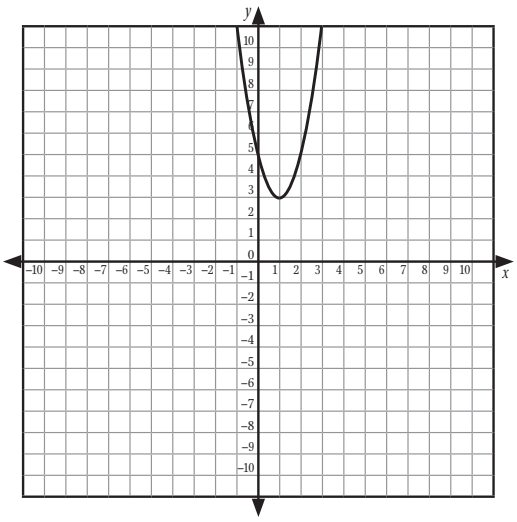
**Station 3**

Given equations in the form $y = (x - h)^2 + k$, students graph parabolas. Students find the $y$-intercept and the axis of symmetry from both the graph and the equation, and begin working towards an understanding of the vertex of a parabola.

**Answers**

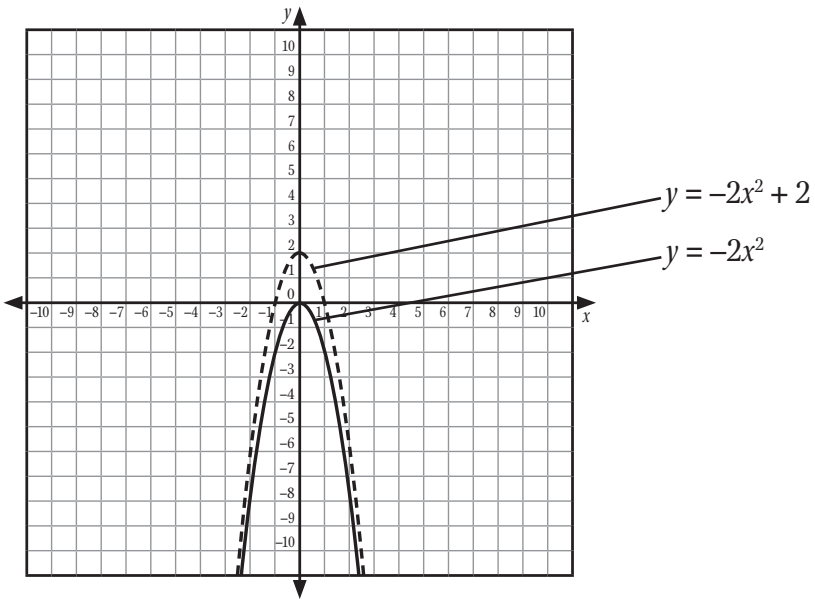
1.

| $x$ | $y$ |
|---|---|
| 0 | 5 |
| 1 | 3 |
| 2 | 5 |
| 3 | 11 |
| 4 | 21 |
| –1 | 11 |
| –2 | 21 |
| –3 | 35 |

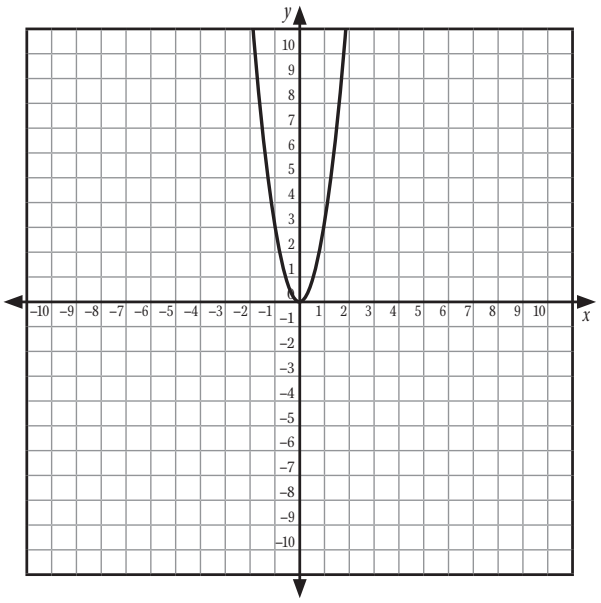



2. $x = 1$
3. (0, 5)

5–6.



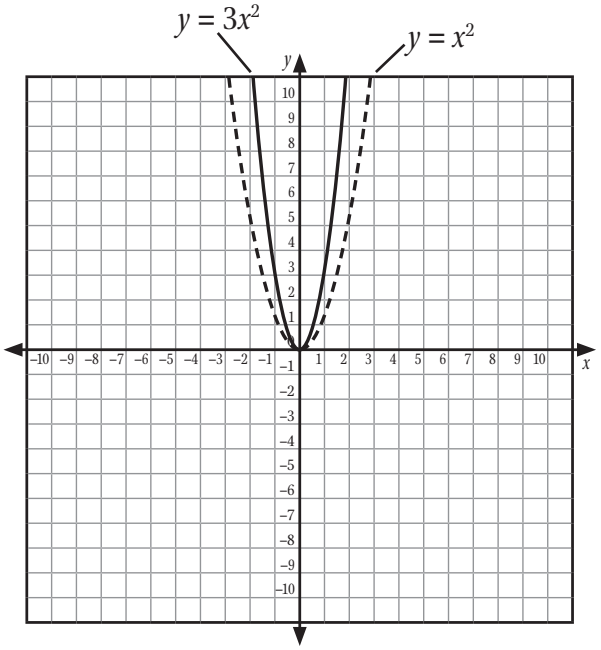

7. (0, 2)
8. The parabola moves vertically.
9. The parabola changes in width.



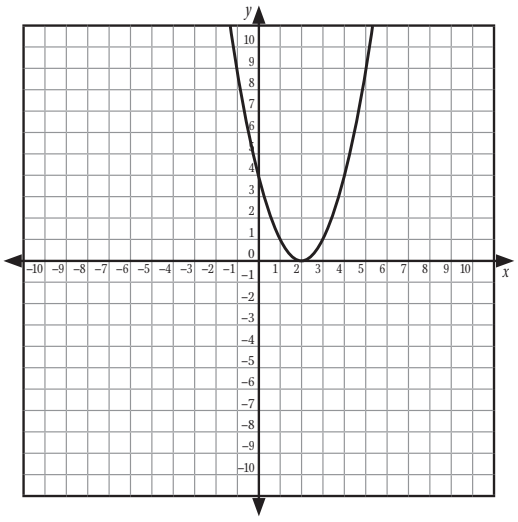

2. (0, 0)

3. $x = 0$

4.




The parabola $3x^2$ is narrower than the parabola $x^2$.

8. 

9. $x = 2$

10. $x = -3$

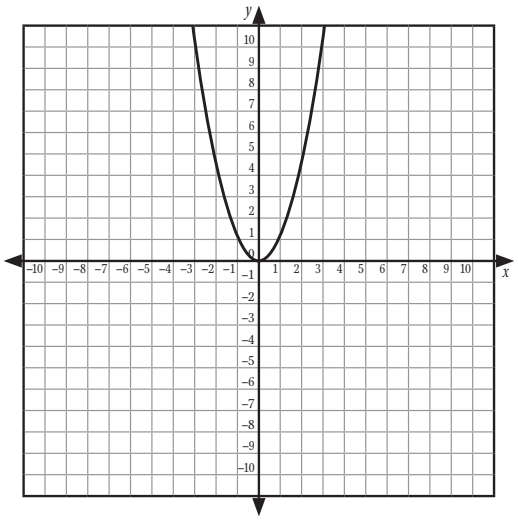
### Station 2

Given equations in the form $y = ax^2$, students graph parabolas. Students compare graphs to explore the relationship between the coefficient of $x$ and the width of the parabola.

### Answers

1.

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 1 | 3 |
| 2 | 12 |
| 3 | 27 |
| –1 | 3 |
| –2 | 12 |
| –3 | 27 |

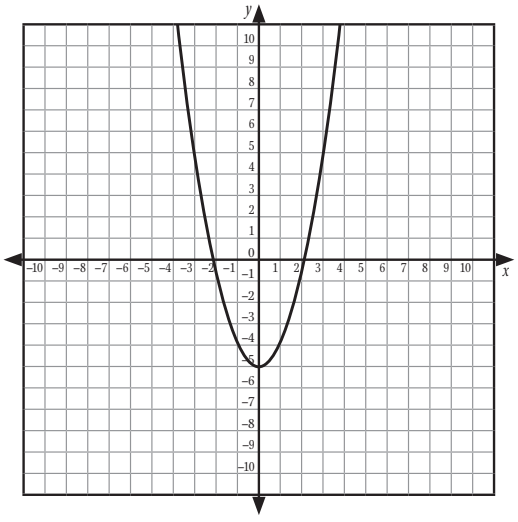
2. 

3. (0, 0)

4. (0, 2)

5. $y = x^2 + 2$

6. $y = x^2 - 5$

7. 

# Quadratics

## Set 1: Quadratic Transformations in Vertex Form

**Instruction**

Goal: To provide opportunities for students to analyze the relationship between the equation of a parabola and its graph

**NCTM Standards**

**Algebra**

Understand patterns, relations, and functions.

Generalize patterns using explicitly defined and recursively defined functions.

- Understand relations and functions and select, convert flexibly among, and use various representations for them.
- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.

Represent and analyze mathematical situations and structures using algebraic symbols.

- Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

## Student Activities Overview and Answer Key

### Station 1

Given equations in the form $y = x^2 + k$ and $y = (x - h)^2$, where $h$ and $k$ are integers, students graph a series of parabolas, finding the $y$-intercept and the axis of symmetry. They explore the relationship between the value of $h$ and $k$ and the position of the parabola with respect to the $x$- and $y$-axes. Students should also begin to understand the relationship between the equation of the parabola and the axis of symmetry.

**Answers**

1.

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |
| –1 | 1 |
| –2 | 4 |
| –3 | 9 |

### Debriefing the Activities

After each group has rotated through every station, bring students together for a brief class discussion. At this time, you might have the groups' spokespersons pose any questions they had about the activities. Before responding, ask if students in other groups encountered the same difficulty or if they have a response to the question. The class discussion is also a good time to reinforce the essential ideas of the activities. The questions that are provided in the teacher's notes for each activity set can serve as a guide to initiating this type of discussion.

You may want to collect the Student Activity Sheets before beginning the class discussion. However, it can be beneficial to collect the sheets afterward so that students can refer to them during the discussion. This also gives students a chance to revisit and refine their work based on the debriefing session.

## Materials List

- 12 index cards
- 6-sided number cube
- algebra tiles
- calculators
- colored pens or pencils
- graph paper
- graphing calculators
- *optional*: computer station with graphing software
- rulers
- *z*-score charts

### Assigning Roles to Students

Students often work most productively in groups when each student has an assigned role. You may want to assign roles to students when they are assigned to groups and change the roles occasionally. Some possible roles are as follows:

- Reader—reads the steps of the activity aloud
- Facilitator—makes sure that each student in the group has a chance to speak and pose questions; also makes sure that each student agrees on each answer before it is written down
- Materials Manager—handles the materials at the station and makes sure the materials are put back in place at the end of the activity
- Timekeeper—tracks the group's progress to ensure that the activity is completed in the allotted time
- Spokesperson—speaks for the group during the debriefing session after the activities

### Timing the Activities

The activities in this book are designed to take approximately 15 minutes per station. Therefore, you might plan on having groups change stations every 15 minutes, with a two-minute interval for moving from one station to the next. It is helpful to give students a "5-minute warning" before it is time to change stations.

Since the activity sets consist of four stations, the above time frame means that it will take about an hour and 10 minutes for groups to work through all stations. If this is followed by a 20-minute class discussion as described below, an entire activity set can be completed in about 90 minutes.

### Guidelines for Students

Before starting the first activity set, you may want to review the following "ground rules" with students. You might also post the rules in the classroom.

- All students in a group should agree on each answer before it is written down. If there is a disagreement within the group, discuss it with one another.
- You can ask your teacher a question only if everyone in the group has the same question.
- If you finish early, work together to write problems of your own that are similar to the ones on the Student Activity Sheet.
- Leave the station exactly as you found it. All materials should be in the same place and in the same condition as when you arrived.

# Introduction

This book includes a collection of station-based activities to provide students with opportunities to practice and apply the mathematical skills and concepts they are learning. It contains sets of activities for Quadratics; Arithmetic Sequences; Functions; Geometric Sequences; Matrices; Conics; and Data Analysis and Probability. You may use these activities in addition to direct instruction, or instead of direct instruction in areas where students understand the basic concepts but need practice. The Discussion Guide included with each set of activities provides an important opportunity to help students reflect on their experiences and synthesize their thinking. It also provides guidance for ongoing, informal assessment to inform instructional planning.

## Implementation Guide

The following guidelines will help you prepare for and use the activity sets in this book.

### Setting Up the Stations

Each activity set consists of four stations. Set up each station at a desk, or at several desks pushed together, with enough chairs for a small group of students. Place a card with the number of the station on the desk. Each station should also contain the materials specified in the teacher's notes, and a stack of Student Activity Sheets (one copy per student). Place the required materials (as listed) at each station.

When a group of students arrives at a station, each student should take one of the activity sheets to record the group's work. Although students should work together to develop one set of answers for the entire group, each student should record the answers on his or her own activity sheet. This helps keep students engaged in the activity and gives each student a record of the activity for future reference.

### Forming Groups of Students

All activity sets consist of four stations. You might divide the class into four groups by having students count off from 1 to 4. If you have a large class and want to have students working in small groups, you might set up two identical sets of stations, labeled A and B. In this way, the class can be divided into eight groups, with each group of students rotating through the "A" stations or "B" stations.

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# Advanced Algebra Station Activities

## for NCTM Standards





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