STATE COMMON CORE STANDARDS

Problem-Based Tasks for Mathematics II

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Introduction

Welcome to *Common Core State Standards Problem-Based Tasks for Mathematics II*. Use these engaging real-world scenarios to help infuse your mathematics program with a problem-based approach to the knowledge and skills required by the Common Core State Standards for Mathematics.

This collection of tasks addresses all of the Common Core State Standard conceptual categories for high school mathematics:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

The tasks support students in developing and using the Mathematical Practices that are a fundamental part of the CCSS. You can implement these tasks flexibly—to walk students through the application of the standard, prior to traditional instruction, or at the end of instruction. The tasks are appropriate for any Mathematics II class, or for other CCSS-based Grade 10 courses.

Each Problem-Based Task is set in a meaningful real-world context to engage student interest and reinforce the relevance of mathematics. Each is tightly aligned to one or more specific standards from the High School CCSS for Mathematics II. Each task combines the specific content of one or more Common Core State Standards with higher-order thinking. Optional coaching questions scaffold the tasks and guide students in solving the problems. Answers and suggested responses to the coaching questions are provided.

Student pages identify the targeted Common Core State Standard(s) and present the problem-solving tasks in familiar and intriguing contexts, and require collaboration, problem solving, reasoning, and communication. You may choose to assign the tasks with little scaffolding (by forgoing use of the coaching questions), or with the series of coaching questions that currently follow each task to lead students through the important steps of the problem. You can also differentiate to meet the needs of individual students by providing coaching questions as appropriate.

We developed these Problem-Based Tasks at the request of math educators and with advice and feedback from mathematics supervisors and teachers. Please let us know how they work in your classroom. We'd love suggestions for improving the tasks, or topics and contexts for creating additional tasks. Visit us at www.walch.com, follow us on Twitter (@WalchEd), or e-mail suggestions to customerservice@walch.com.

Standards Correlations

Common Core State Standards Problem-Based Tasks for Mathematics II is correlated to the Common Core State Standards for high school mathematics. The table that follows lists each task's targeted Common Core State Standard(s), focus, title, and starting page number. The materials are organized in the order in which the standards are listed in the mathematics CCSS, and are grouped by high school conceptual category. Stars (★) indicate modeling standards. To access the full text of the Common Core State Standards for high school mathematics, view the PDF found at http://www.walch.com/CCSS/00001.

CCSS addressed	Task focus	Task title	Page number
Number and Quantity: The Real Number System			
N–RN.1 N–RN.2	Defining, Rewriting, and Evaluating Rational Exponents	Population Growth	1
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Number and Quantity: The Complex Number System			
N–CN.1	Defining Complex Numbers, i, and i^2	Representing Impedance	8
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A–SSE.1a★	Identifying Terms, Factors, and Coefficients	Deck the Deck	27
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A–CED.2★ A–SSE.3a★	Creating and Graphing Equations Using Standard Form	Parabolic Party Streamers	34

(*continued*)

N–RN.1; N–RN.2 • Number and Quantity

Defining, Rewriting, and Evaluating Rational Exponents

Common Core State Standards

N–RN.1

Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*

N–RN.2

Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Problem-Based Task: Population Growth

A town takes a census, or a count of its population, every 10 years. The town uses the census to estimate the population's growth rate. The town's population today is 42,000 people, and the 10-year growth rate is approximately 35%. The town's population can be estimated at any year t using the equation $y = y_0 \bullet (1+r)^{\frac{t}{10}}$, where y_0 is the initial population and r is the 10-year growth rate. What will be the town's approximate population 8 years from today?

Problem-Based Task: Population Growth

Coaching

a. What equation can you use to estimate the population in this situation?

b. What do each of the variables in the equation represent?

c. Which variables can you replace with quantities, and what are the quantities?

d. How can you further simplify the equation before you estimate the population?

e. What will be the town's approximate population 8 years from today?

Problem-Based Task: Population Growth

Coaching Sample Responses

a. What equation can you use to estimate the population in this situation?

$$y = y_0 \bullet (1+r)^{\frac{t}{10}}$$

b. What do each of the variables in the equation represent?

y is the population in 8 years, y_0 is the population today, r is the 10-year growth rate, and t is the time in years.

c. Which variables can you replace with quantities, and what are the quantities?

y_0, r, and t are all given. y_0 is 42,000, r is 0.35, and t is 8.

d. How can you further simplify the equation before you estimate the population?

Write the original equation with quantities in place of variables.

$$y = 42{,}000 \bullet (1+0.35)^{\frac{8}{10}}$$

The rational exponent can be reduced to simplest form. Note that the original root is even; therefore, the solution must be positive. Because all values in the equation are positive, no absolute value symbols are needed.

$$y = 42{,}000 \bullet (1+0.35)^{\frac{8}{10}}$$

$$y = 42{,}000 \bullet (1+0.35)^{\frac{4}{5}}$$

e. What will be the town's approximate population 8 years from today?

Use a calculator to find the town's approximate population 8 years from today.

$$y = 42{,}000 \bullet (1+0.35)^{\frac{4}{5}}$$

$$y = 42{,}000 \bullet (1.35)^{\frac{4}{5}}$$

$$y = 42{,}000 \bullet (1.0271)$$

$$y \approx 53{,}396.9$$

The town's approximate population will be 53,397 people 8 years from today.

Common Core State Standards

N–RN.2

Rewrite expressions involving radicals and rational exponents using the properties of exponents.

N–RN.3

Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Problem-Based Task: Estimating Depreciation

Yasmina is buying a new car. To estimate how her car will decrease in value, or depreciate, she looks at the price of older versions of the same car. She finds a similar car that is 2.5 years old. The original price of the car was $22,000, and the current selling price is $16,905. She knows the equation $c = 22{,}000 \bullet d^t$ can be used to estimate the value of the car in any year t after being purchased for $22,000. The value d is used to calculate the new value of the car each year. Write an equation to help Yasmina estimate the value of her new car, c, in any year t, if the original purchase price is $22,000.

Problem-Based Task: Estimating Depreciation

Coaching

a. Which variable needs to be determined in the equation $c = 22{,}000 \bullet d^t$ so that Yasmina can estimate the value of a car in year t?

b. Use the older car to help Yasmina complete the equation. Which variables are known for this car?

c. Substitute the known quantities for the variables in the equation.

d. Use inverse operations to find an equivalent equation in the form $x^a = b$, where x is the only variable.

e. Solve for the variable needed to complete the equation.

f. What is the equation Yasmina can use to estimate the value of a car purchased for $22,000 in any year t?

g. What will be the value of her new car 6 years after she has purchased it?

Problem-Based Task: Estimating Depreciation

Coaching Sample Responses

a. Which variable needs to be determined in the equation $c = 22{,}000 \bullet d^t$ so that Yasmina can estimate the value of a car in year *t*?

The variable *c* represents the value of the car in year *t*; *t* is the independent variable and *c* is the dependent variable. The only variable that needs to be determined is *d*.

b. Use the older car to help Yasmina complete the equation. Which variables are known for this car?

For the older car, Yasmina knows the current value, *c*, and the time, *t*, in years.

c. Substitute the known quantities for the variables in the equation.

$c = \$16{,}905$

$t = 2.5$ years

$16{,}905 = 22{,}000 \bullet d^{2.5}$

d. Use inverse operations to find an equivalent equation in the form $x^a = b$, where *x* is the only variable.

$16{,}905 = 22{,}000 \bullet d^{2.5}$

$0.768 \approx d^{2.5}$

e. Solve for the variable needed to complete the equation.

$0.768 \approx d^{2.5}$

$0.768 \approx d^{2.5}$

$0.768 \approx d^{\frac{5}{2}}$

$(0.768)^{\frac{2}{5}} \approx \left(d^{\frac{5}{2}}\right)^{\frac{2}{5}}$

$0.90 \approx d$

f. What is the equation Yasmina can use to estimate the value of a car purchased for $22,000 in any year t?

Replace d with the calculated value of 0.90.

$c = 22{,}000(0.90)^t$

g. What will be the value of her new car 6 years after she has purchased it?

Replace t with 6 in the equation found in part f.

$c = 22{,}000(0.90)^6$

$c = 11{,}691.70$

After 6 years, the car will be worth approximately $11,692, or about half of its original value.

Common Core State Standard

N–CN.1

Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

Problem-Based Task: Representing Impedance

Impedance is the measure of an object's resistance to an electric current, or its opposition to the flow of a current. Complex numbers are used to represent the impedance of an element in a circuit. A certain element has a voltage of 21 volts and a current of 1.25 milliamperes. If impedance is equal to $V + Ii$, where I is in milliamperes, then what is the element's impedance?

Problem-Based Task: Representing Impedance

Coaching

a. Which quantity will be the real part of the complex number?

b. Which quantity will be the multiple of *i* in the complex number?

c. How is the multiple of *i* used to write the imaginary part of a complex number?

d. How are the real and imaginary parts used to write a complex number?

e. What is the element's impedance?

Problem-Based Task: Representing Impedance

Coaching Sample Responses

a. Which quantity will be the real part of the complex number?

The voltage, 21, is the real part of the complex number.

b. Which quantity will be the multiple of i in the complex number?

The current, 1.25, is the multiple of i in the complex number.

c. How is the multiple of i used to write the imaginary part of a complex number?

The product of the multiple of i and i is the imaginary part: $1.25 \bullet i = 1.25i$.

d. How are the real and imaginary parts used to write a complex number?

The sum of the real and imaginary parts is a complex number.

$21 + 1.25i$

e. What is the element's impedance?

The element's impedance is $21 + 1.25i$.

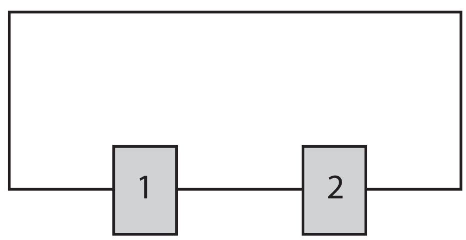
Common Core State Standard

N–CN.2

Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Problem-Based Task: Elements in Series in a Circuit

The impedance of an element can be represented using the complex number, $V + Ii$, where V is the element's voltage and I is the element's current in milliamperes. If two elements are in a circuit in series, the total impedance is the sum of the impedance of each element. The following diagram of a circuit contains two elements, 1 and 2, in series.



Element 1 has a voltage of 30.5 volts and a current of 2.8 milliamperes. Element 2 has a voltage of 19 volts and a current of 3 milliamperes. What is the total impedance of the circuit?

Problem-Based Task: Elements in Series in a Circuit

Coaching

a. How can the voltage and current be used to write a complex number to represent the impedance of each element?

b. What is the impedance of each element?

c. What operation is used to find the total impedance if the elements are in series?

d. Describe how to find the sum or difference of two complex numbers by using their real and imaginary parts.

e. What is the total impedance of the circuit?

Problem-Based Task: Elements in Series in a Circuit

Coaching Sample Responses

a. How can the voltage and current be used to write a complex number to represent the impedance of each element?

The impedance of each element can be written in the form $V + Ii$, where V is the voltage and I is the current in milliamperes.

b. What is the impedance of each element?

Element 1: $30.5 + 2.8i$

Element 2: $19 + 3i$

c. What operation is used to find the total impedance if the elements are in series?

Addition is used to find total impedance.

Therefore, find the sum of the impedance of each element.

d. Describe how to find the sum or difference of two complex numbers by using their real and imaginary parts.

To find the sum of two complex numbers, find the sum of the real parts, and find the multiple of i by summing the coefficients of i from each complex number.

To find the difference of two complex numbers, find the difference of the real parts, and find the multiple of i by subtracting the coefficients of i from each complex number.

e. What is the total impedance of the circuit?

$(30.5 + 2.8i) + (19 + 3i)$

$= (30.5 + 19) + (2.8 + 3)i$

$= 49.5 + 5.8i$

The total impedance of the circuit is $49.5 + 5.8i$.

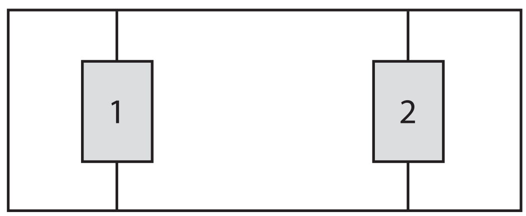
Common Core State Standard

N–CN.2

Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Problem-Based Task: Elements in Parallel in a Circuit

The impedance of an element can be represented using the complex number, $V + Ii$, where V is the element's voltage and I is the element's current in milliamperes. The following diagram of a circuit contains two elements, 1 and 2, in parallel.



If the impedance of element 1 is $Z_1 = 15 + i$, and the impedance of element 2 is $Z_2 = 10 + 2i$, the total impedance of the two elements in parallel is $\frac{1}{Z_1} + \frac{1}{Z_2}$. What is the total impedance for the two elements in parallel? Leave your response as a fraction.

Problem-Based Task: Elements in Parallel in a Circuit

Coaching

a. What is the reciprocal of each impedance?

b. How can a common denominator be calculated?

c. Find the common denominator.

d. Use the factor needed to calculate the common denominator to write an equivalent reciprocal for each impedance.

e. What is the sum of these reciprocals, written as a fraction?

f. What is the total impedance for the two elements in parallel? Leave your response as a fraction.

Problem-Based Task: Elements in Parallel in a Circuit

Coaching Sample Responses

a. What is the reciprocal of each impedance?

$$\frac{1}{Z_1}=\frac{1}{15+i} \qquad \frac{1}{Z_2}=\frac{1}{10+2i}$$

b. How can a common denominator be calculated?

The product of the two existing denominators can be used as the common denominator.

c. Find the common denominator.

Multiply both terms in the first polynomial by both terms in the second polynomial. Find the product of the first terms, outside terms, inside terms, and last terms.

$(15 + i)(10 + 2i)$

$= 150 + 30i + 10i + 2i^2$

$= 148 + 40i$

d. Use the factor needed to calculate the common denominator to write an equivalent reciprocal for each impedance.

$$\frac{1}{Z_1}=\frac{1}{15+i}=\frac{1}{15+i}\bullet\frac{10+2i}{10+2i}=\frac{10+2i}{148+40i}$$

$$\frac{1}{Z_2}=\frac{1}{10+2i}=\frac{1}{10+2i}\bullet\frac{15+i}{15+i}=\frac{15+i}{148+40i}$$

e. What is the sum of these reciprocals, written as a fraction?

$$\frac{10+2i}{148+40i}+\frac{15+i}{148+40i}=\frac{25+3i}{148+40i}$$

f. What is the total impedance for the two elements in parallel? Leave your response as a fraction.

The total impedance for the two elements in parallel is $\frac{25+3i}{148+40i}$.

Common Core State Standards

N–CN.7

Solve quadratic equations with real coefficients that have complex solutions.

N–CN.9

(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Problem-Based Task: All Roads Lead to the Same Destination

Solve $3x^2 + 5 = 0$ by the following three methods. Show that all three methods give the same solutions.

- Use the quadratic formula.
- Use this property of square roots: If $x^2 = a$, then $x = \pm\sqrt{a}$.
- Show through factoring that $3x^2 + 5$ can be written as a sum of two squares.

Problem-Based Task: All Roads Lead to the Same Destination

Coaching

a. The quadratic formula states that if $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. What are the values of a, b, and c in the equation $3x^2 + 5 = 0$?

b. How can you solve the equation with the quadratic formula?

c. Recall that if $x^2 = a$, then $x = \pm\sqrt{a}$. How can you solve the equation by using this property of square roots? How can you show that your solutions by this method match the solutions in part b?

d. How can you solve the equation by factoring? (*Hint*: Write the given equation. Then write the equation again, showing that $3x^2 + 5$ is the sum of two squares.)

e. How can you apply this identity that shows how to factor the sum of two squares: $a^2 + b^2 = (a + bi)(a - bi)$?

f. The Zero Product Property allows you to state that if $(a + bi)(a - bi) = 0$, then either $a + bi = 0$ or $a - bi = 0$. How can you use the Zero Product Property to finish solving and then write your solutions in the form that matches the solutions in parts b and c?

Problem-Based Task: All Roads Lead to the Same Destination

Coaching Sample Responses

a. The quadratic formula states that if $ax^2 + bx + c = 0$, then $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. What are the values of a, b, and c in the equation $3x^2 + 5 = 0$?

Since the equation is already written in standard form, $a = 3$, $b = 0$, and $c = 5$.

b. How can you solve the equation by the quadratic formula?

$3x^2 + 5 = 0$	Write the given equation.
$x=\frac{-0\pm\sqrt{0^2-4(3)(5)}}{2(3)}$	Substitute values for a, b, and c: $a = 3$, $b = 0$, $c = 5$.
$x=\frac{\pm\sqrt{-60}}{6}$	Simplify.
$x=\frac{\pm i\sqrt{60}}{6}$	For any positive real number a, $\sqrt{-a}=i\sqrt{a}$.
$x=\frac{\pm 2i\sqrt{15}}{6}$	Simplify.
$x=\pm\left(\frac{\sqrt{15}}{3}\right)i$	

c. Recall that if $x^2 = a$, then $x=\pm\sqrt{a}$. How can you solve the equation by using this property of square roots? How can you show that your solutions by this method match the solutions in part b?

$3x^2 + 5 = 0$	Write the given equation.
$3x^2 = -5$	Subtract 5 from both sides.
$x^2=-\frac{5}{3}$	Divide both sides by 3.
$x=\pm\sqrt{-\frac{5}{3}}$	Apply this property of square roots: If $x^2 = a$, then $x=\pm\sqrt{a}$.

$x=\pm i\sqrt{\frac{5}{3}}$	For any positive real number a, $\sqrt{-a}=i\sqrt{a}$.
$x=\pm i\frac{\sqrt{5}}{\sqrt{3}}$	For any real numbers a and b with $b\neq 0$, $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$.
$x=\pm i\frac{\sqrt{5}}{\sqrt{3}}\bullet\frac{\sqrt{3}}{\sqrt{3}}$	Multiply by $\frac{\sqrt{3}}{\sqrt{3}}$, which is equal to 1.
$x=\pm i\frac{\sqrt{5\bullet 3}}{\sqrt{3\bullet 3}}$	For any real numbers a and b, $\sqrt{a}\bullet\sqrt{b}=\sqrt{ab}$.
$x=\pm i\frac{\sqrt{15}}{\sqrt{9}}$	Simplify.
$x=\pm i\frac{\sqrt{15}}{3}$	
$x=\pm\left(\frac{\sqrt{15}}{3}\right)i$	

d. How can you solve the equation by factoring? (*Hint*: Write the given equation. Then write the equation again, showing that $3x^2 + 5$ is the sum of two squares.)

$3x^2 + 5 = 0$	Write the given equation.
$\left(x\sqrt{3}\right)^2+\left(\sqrt{5}\right)^2=0$	Show that $3x^2 + 5$ is the sum of two squares.

e. How can you apply this identity that shows how to factor the sum of two squares: $a^2 + b^2 = (a + bi)(a - bi)$?

$3x^2 + 5 = 0$	Given equation
$\left(x\sqrt{3}\right)^2+\left(\sqrt{5}\right)^2=0$	Show that $3x^2 + 5$ is the sum of two squares.
$\left(x\sqrt{3}+i\sqrt{5}\right)\left(x\sqrt{3}-i\sqrt{5}\right)=0$	Apply the identity $a^2 + b^2 = (a + bi)(a - bi)$.

f. The Zero Product Property allows you to state that if $(a + bi)(a - bi) = 0$, then either $a + bi = 0$ or $a - bi = 0$. How can you use the Zero Product Property to finish solving and then write your solutions in the form that matches the solutions in parts b and c?

Start with the factorization found in part e.

$$\left(x\sqrt{3}+i\sqrt{5}\right)\left(x\sqrt{3}-i\sqrt{5}\right)=0 \qquad a^2 + b^2 = (a + bi)(a - bi)$$

$$x\sqrt{3}+i\sqrt{5}=0 \text{ or } x\sqrt{3}-i\sqrt{5}=0 \qquad \text{If } (a + bi)(a - bi) = 0\text{, then either } a + bi = 0 \text{ or } a - bi = 0.$$

$$x\sqrt{3}=-i\sqrt{5} \text{ or } x\sqrt{3}=i\sqrt{5}$$

$$x=-\frac{\sqrt{5}}{\sqrt{3}}i \text{ or } x=\frac{\sqrt{5}}{\sqrt{3}}i$$

$$x=-\frac{\sqrt{5}}{\sqrt{3}}\bullet\frac{\sqrt{3}}{\sqrt{3}}i \text{ or } x=\frac{\sqrt{5}}{\sqrt{3}}\bullet\frac{\sqrt{3}}{\sqrt{3}}i$$

$$x=-\left(\frac{\sqrt{15}}{3}\right)i \text{ or } x=\left(\frac{\sqrt{15}}{3}\right)i$$

$$x=\pm\left(\frac{\sqrt{15}}{3}\right)i$$

N–CN.8(+) • Number and Quantity

Extending Polynomial Identities to Include Complex Numbers

Common Core State Standard

N–CN.8

(+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.

Problem-Based Task: Making a Complex Expression Simple

Kiana is working on a homework problem and has to simplify the expression $\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^3$. She must then complete the following equation and explain why it is an identity: $\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^3 \bullet a = \_\_\_$.