COMMON CORE STATE STANDARDS

Problem-Based Tasks

for Mathematics III

WALCH EDUCATION®

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Introduction

Welcome to *Common Core State Standards Problem-Based Tasks for Mathematics III.* Use these engaging real-world scenarios to help infuse your mathematics program with a problem-based approach to the knowledge and skills required by the Common Core State Standards for Mathematics.

This collection of tasks addresses all of the Common Core State Standard conceptual categories for high school mathematics:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

The tasks support students in developing and using the Mathematical Practices that are a fundamental part of the CCSS. You can implement these tasks flexibly—to walk students through the application of the standard, prior to traditional instruction, or at the end of instruction. The tasks are appropriate for any Mathematics III class, or for other CCSS-based Grade 11 courses.

Each Problem-Based Task is set in a meaningful real-world context to engage student interest and reinforce the relevance of mathematics. Each is tightly aligned to one or more specific standards from the High School CCSS for Mathematics III. Each task combines the specific content of one or more Common Core State Standards with higher-order thinking. Optional coaching questions scaffold the tasks and guide students in solving the problems. Answers and suggested responses to the coaching questions are provided.

Student pages identify the targeted Common Core State Standard(s) and present the problem-solving tasks in familiar and intriguing contexts, and require collaboration, problem solving, reasoning, and communication. You may choose to assign the tasks with little scaffolding (by forgoing use of the coaching questions), or with the series of coaching questions that currently follow each task to lead students through the important steps of the problem. You can also differentiate to meet the needs of individual students by providing coaching questions as appropriate.

We developed these Problem-Based Tasks at the request of math educators and with advice and feedback from mathematics supervisors and teachers. Please let us know how they work in your classroom. We'd love suggestions for improving the tasks, or topics and contexts for creating additional tasks. Visit us at www.walch.com, follow us on Twitter (@WalchEd), or e-mail suggestions to customerservice@walch.com.

Standards Correlations

Common Core State Standards Problem-Based Tasks for Mathematics III is correlated to the Common Core State Standards for high school mathematics. The table that follows lists each task's targeted Common Core State Standard(s), focus, title, and starting page number. The materials are organized in the order in which the standards are listed in the mathematics CCSS, and are grouped by high school conceptual category. Stars (★) indicate modeling standards. To access the full text of the Common Core State Standards for high school mathematics, view the PDF found at http://www.walch.com/CCSS/00001.

CCSS addressed	Task focus	Task title	Page number
Number and Quantity: The Complex Number System			
N–CN.8 (+) A–SSE.1b★ A–SSE.2 A–APR.4	Complex Polynomial Identities	Measuring Electrical Impedance	1
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A–APR.1	Adding and Subtracting Polynomials	Garden Perimeter	19
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Standards Correlations

CCSS addressed	Task focus	Task title	Page number
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A–SSE.2 A–APR.7 (+)	Multiplying Rational Expressions	The Area of a Triangle	44
A–SSE.2 A–APR.6 A–APR.7 (+)	Dividing Rational Expressions	Fuel Economy	48
Algebra: Creating Equations			
A–CED.1★	Creating Equations in One Variable	How Long Will the Fertilizer Last?	51
A–CED.2★ F–IF.4★ F–IF.5★ F–BF.3	Linear, Exponential, and Quadratic Functions	Comparing Social Media Growth	57
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A–REI.2	Solving Rational Equations	Snow Removal	72
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A–REI.11★	Solving Systems of Equations Graphically	Advertise Here!	86

(*continued*)

Name: **Date:**

N–CN.8(+); A–SSE.1b★; A–SSE.2; A–APR.4 • Number and Quantity

Complex Polynomial Identities

Common Core State Standards

N–CN.8

(+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.

A–SSE.1

Interpret expressions that represent a quantity in terms of its context.★

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1 + r)^n$ *as the product of P and a factor not depending on P.*

A–SSE.2

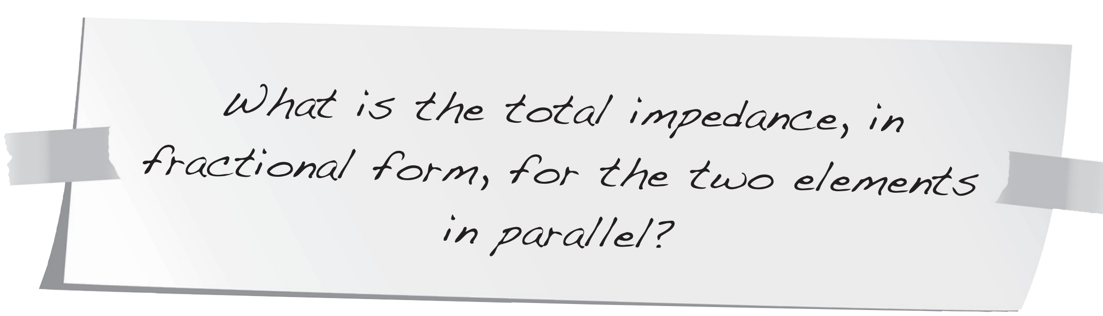
Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

A–APR.4

Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*

Problem-Based Task: Measuring Electrical Impedance

Impedance measures the total opposition that a circuit presents to an electric current. The impedance of an element can be represented using a complex number $V + Ii$, where V is the element's voltage and I is the element's current. If the impedance of Element 1 is $Z_1 = 12 + i$, and the impedance of Element 2 is $Z_2 = 14 + i$, the total impedance of the two elements in parallel is $\frac{1}{Z_1} + \frac{1}{Z_2}$. What is the total impedance, in fractional form, for the two elements in parallel?



Problem-Based Task: Measuring Electrical Impedance

Coaching

a. What is the reciprocal of each element's impedance?

b. What is the conjugate of each denominator?

c. Rationalize each denominator using a complex conjugate.

d. What is the total impedance, in fractional form, for the two elements in parallel?

Problem-Based Task: Measuring Electrical Impedance

Coaching Sample Responses

a. What is the reciprocal of each element's impedance?

The impedance of Element 1 is $Z_1 = 12 + i$.

The reciprocal of the impedance of Element 1 is $\frac{1}{Z_1} = \frac{1}{12+i}$.

The impedance of Element 2 is $Z_2 = 14 + i$.

The reciprocal of the impedance of Element 2 is $\frac{1}{Z_2} = \frac{1}{14+i}$.

b. What is the conjugate of each denominator?

The conjugate of a rational number $a + bi$ is $a - bi$.

The conjugate of the denominator $12 + i$ is $12 - i$.

The conjugate of the denominator $14 + i$ is $14 - i$.

c. Rationalize each denominator using a complex conjugate.

To rationalize a denominator, the imaginary unit i must be removed from the denominator.

The product of a complex number and its conjugate is a real number.

Multiply the numerator and denominator of each fraction by the conjugate.

Element 1

$$\frac{1}{Z_1} = \frac{1}{12+i} \bullet \frac{12-i}{12-i}$$

$$= \frac{12-i}{12^2+1}$$

$$= \frac{12-i}{145}$$

Element 2

$$\frac{1}{Z_2} = \frac{1}{14+i} \bullet \frac{14-i}{14-i}$$

$$= \frac{14-i}{14^2+1}$$

$$= \frac{14-i}{197}$$

d. What is the total impedance, in fractional form, for the two elements in parallel?

To add the two fractions, first find a common denominator using the product of the two denominators.

Element 1

$$\frac{1}{Z_1}=\frac{12-i}{145}\bullet\frac{197}{197}$$

$$=\frac{2364-197i}{28{,}565}$$

Element 2

$$\frac{1}{Z_2}=\frac{14-i}{197}\bullet\frac{145}{145}$$

$$=\frac{2030-145i}{28{,}565}$$

Add the two reciprocals with the rationalized denominators.

$$\frac{1}{Z_1}+\frac{1}{Z_2}=\frac{2364-197i}{28{,}565}+\frac{2030-145i}{28{,}565}$$

$$=\frac{4394-342i}{28{,}565}$$

The total impedance in fractional form for the two elements in parallel is $\frac{4394-342i}{28{,}565}$.

Name: **Date:**

N–CN.8(+); A–SSE.1b★; A–SSE.2; A–APR.4 • Number and Quantity

Complex Polynomial Identities

Common Core State Standards

N–CN.8

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The total impedance in fractional form for the two elements in parallel is $\frac{4394-342i}{28{,}565}$.

Name: **Date:**

A–SSE.1a★ • Algebra

Structures of Expressions

Common Core State Standard

A–SSE.1

Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

Problem-Based Task: Laying Tile

A contractor is creating a design using different-sized rectangular tiles. The area of each tile is shown in the diagram below. What is the total area of the shown strip of tile? (Diagram not shown to scale.)

36 in^2	$3x$ in^2	x^2 in^2

Name: **Date:**

Common Core State Standard

A–SSE.1

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Problem-Based Task: Laying Tile

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36 in^2	3x in^2	x^2 in^2

Problem-Based Task: Laying Tile

Coaching

a. How do you find the total area of the strip of tile?

b. What expression can be written to show the total area of the strip of tile?

c. Typically, in what order are the terms in a polynomial expression written?

d. What is the polynomial expression that shows the total area of the strip of tile?

Problem-Based Task: Laying Tile

Coaching Sample Responses

a. How do you find the total area of the strip of tile?

To find the total area, add together the areas of each component.

b. What expression can be written to show the total area of the strip of tile?

The areas of each component of the strip of tile are 36 in^2, $3x$ in^2, and x^2 in^2. The total area is the sum of these three smaller areas, or $(36 + 3x + x^2)$ in^2.

c. Typically, in what order are the terms in a polynomial expression written?

In a polynomial expression, the terms are listed in descending order based on the power of the variable; the term with the highest power is listed first, followed by the term with the next highest power, and so on. The constant is listed last since it is only a numeric quantity.

d. What is the polynomial expression that shows the total area of the strip of tile?

The term with the highest power is x^2, so it's listed first; then $3x$; the constant, 36, is last.

Therefore, the total area can be represented by the expression $(x^2 + 3x + 36)$ in^2.

A–SSE.1a★; A–SSE.1b★; A–SSE.2 • Algebra

Structures of Rational Expressions

Common Core State Standards

A–SSE.1

Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.*

A–SSE.2

Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Problem-Based Task: Rewriting a Rational Expression

As part of their preparations for an upcoming math test, Faith, Jia, and Kayla work together to simplify the rational expression $\dfrac{t^4-16}{t^2-t-6}$.

- Faith reasons that, because there is a factor of t^4 in the numerator but a factor of only t^2 in the denominator, there is no way to simplify this expression. She says it is already as simple as it can be. Is Faith's analysis correct?

- After factoring the numerator, Jia says that the expression is equivalent to $\dfrac{(t-2)(t^2+4)}{t-3}$. Is Jia correct?

- Kayla tries a different method. After much factoring, she arrives at the expression $\dfrac{8-4t+2t^2-t^3}{3-t}$. Is Kayla correct?

Is Faith's analysis correct? Is Jia correct? Is Kayla correct?

Problem-Based Task: Rewriting a Rational Expression

Coaching

a. Do you see any familiar pattern in the denominator of the rational expression $\frac{t^4-16}{t^2-t-6}$?

b. If the expression looks like a quadratic, it might be possible to factor it. How could you do that?

c. What is the factored form of the denominator, $t^2 - t - 6$?

d. What is the square of t^2?

e. Is 16 a perfect square?

f. What do the results of parts d and e imply about the numerator of the rational expression $\frac{t^4-16}{t^2-t-6}$? Is this a familiar pattern?

g. What is the factored form of the numerator, $t^4 - 16$?

h. How could you write an equivalent expression using the factored forms of the numerator and denominator?

i. In this factored form, do any factors cancel? If so, what is the resulting expression?

j. What does this result reveal about Faith's answer?

k. How does this result compare to Jia's answer?

l. Does this result have anything in common with Kayla's answer?

m. What happens if you expand the terms of the numerator in the expression from part i—that is, if you rewrite the terms to eliminate the parentheses?

n. How could you adjust the denominator in the factored expression to match the denominator in Kayla's answer?

o. The action taken in part n must also be applied to the numerator in order to maintain an equivalent expression. When you do so, what is the result?

p. What does this result imply about Kayla's answer?

Problem-Based Task: Rewriting a Rational Expression

Coaching Sample Responses

a. Do you see any familiar pattern in the denominator of the rational expression $\frac{t^4-16}{t^2-t-6}$?

The denominator, $t^2 - t - 6$, looks like a quadratic expression.

b. If the expression looks like a quadratic, it might be possible to factor it. How could you do that?

To factor the quadratic, you could use the quadratic formula, try a table or a graph, or try to find factors that work through trial and error.

c. What is the factored form of the denominator, $t^2 - t - 6$?

The factors of $t^2 - t - 6$ are $(t - 3)$ and $(t + 2)$; therefore, the factored form is $(t - 3)(t + 2)$.

d. What is the square of t^2?

$(t^2)^2 = t^4$

e. Is 16 a perfect square?

Yes; $16 = 4^2$.

f. What do the results of parts d and e imply about the numerator of the rational expression $\frac{t^4-16}{t^2+t-6}$? Is this a familiar pattern?

The results of parts d and e imply that $t^4 - 16$ is the difference of two squares. This is a familiar quadratic form.

g. What is the factored form of the numerator, $t^4 - 16$?

As the difference of two squares, $t^4 - 16 = (t^2 - 4)(t^2 + 4)$. The first term is, itself, a difference of two squares and can be factored further. Therefore, $t^4 - 16 = (t - 2)(t + 2)(t^2 + 4)$.

h. How could you write an equivalent expression using the factored forms of the numerator and denominator?

The original expression can be rewritten in factored form as follows.

$$\frac{t^4-16}{t^2-t-6}=\frac{(t-2)(t+2)(t^2+4)}{(t-3)(t+2)}$$

i. In this factored form, do any factors cancel? If so, what is the resulting expression?

Both the numerator and denominator have the factor $t + 2$, which can be factored out as follows.

$$\frac{t^4-16}{t^2-t-6}=\frac{(t-2)\cancel{(t+2)}(t^2+4)}{(t-3)\cancel{(t+2)}}=\frac{(t-2)(t^2+4)}{(t-3)}$$

j. What does this result reveal about Faith's answer?

We were able to factor the expression into a simpler form, so Faith's answer is incorrect.

k. How does this result compare to Jia's answer?

This result matches the expression given by Jia; therefore, Jia is correct.

l. Does this result have anything in common with Kayla's answer?

The expression from part i, $\frac{(t-2)(t^2+4)}{(t-3)}$, looks a little like Kayla's answer of $\frac{8-4t+2t^2-t^3}{3-t}$. There is an expression in the denominator (for which $t \neq 3$). However, the expression from part i is in factored form, whereas Kayla's answer is in expanded form.

m. What happens if you expand the terms of the numerator in the expression from part i—that is, if you rewrite the terms to eliminate the parentheses?

Expanding the numerator results in the following expression.

$(t - 2)(t^2 + 4)$

$= t(t^2 + 4) - 2(t^2 + 4)$

$= t^3 + 4t - 2t^2 - 8$

$= t^3 - 2t^2 + 4t - 8$

This resembles the numerator in Kayla's answer, except it is reversed. Also, compared with Kayla's numerator, every positive term is negative and the negative terms are positive.

n. How could you adjust the denominator in the factored expression to match the denominator in Kayla's answer?

The denominator of Kayla's answer is $3 - t$.

The denominator of the factored expression is $t - 3$.

Multiply $t - 3$ by -1.

$-1(t - 3)$

$= -t + 3$

$= 3 - t$

This matches the denominator of Kayla's expression.

o. The action taken in part n must also be applied to the numerator of the factored expression in order to maintain an equivalent expression. When you do so, what is the result?

From part m, we determined the numerator of the factored expression, when written in expanded form, is $t^3 - 2t^2 + 4t - 8$.

Multiply $t^3 - 2t^2 + 4t - 8$ by -1.

$-1(t^3 - 2t^2 + 4t - 8)$

$= -t^3 + 2t^2 - 4t + 8$

$= 8 - 4t + 2t^2 - t^3$

This is an exact match between our result and Kayla's factored form. Therefore, we can finally see the following:

$$\frac{t^3-2t^2+4t-8}{t-3}=\frac{t^3-2t^2+4t-8}{t-3}\bullet\frac{-1}{-1}=\frac{-(t^3-2t^2+4t-8)}{-1(t-3)}=\frac{8-4t+2t^2-t^3}{3-t}$$

p. What does this result imply about Kayla's answer?

Kayla's answer is correct.

Name: ______________________________ **Date:** ______________

A–SSE.1a★; A–SSE.1b★; A–SSE.2; A–APR.4; A–APR.5(+) • Algebra

The Binomial Theorem

Common Core State Standards

A–SSE.1

Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1 + r)^n$ *as the product of P and a factor not depending on P.*

A–SSE.2

Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

A–APR.4

Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*

A–APR.5

(+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[1]

[1] The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

Problem-Based Task: Combinations of Candidates

There are 9 students running for 4 openings on the student council. Only students who are running for election can be elected; no write-in candidates are allowed. An elected student can only hold one position at a time. Use Pascal's Triangle to find the number of different ways 4 students can be elected from the group of 9 candidates.

Use Pascal's Triangle to find the number of different ways 4 students can be elected from the group of 9 candidates.

Problem-Based Task: Combinations of Candidates

Coaching

a. How many students are running for election?

b. Create Pascal's Triangle to the appropriate row.

c. How many students can be elected?

d. How many different combinations of 4 elected students can be created using the 9 candidates?

Problem-Based Task: Combinations of Candidates

Coaching Sample Responses

a. How many students are running for election?

There are 9 students running for student council.

b. Create Pascal's Triangle to the appropriate row.

Note that the first line of Pascal's Triangle is "row 0," so ten lines of the triangle are needed to find row 9.

Row 0	1									
Row 1	1	1								
Row 2	1	2	1							
Row 3	1	3	3	1						
Row 4	1	4	6	4	1					
Row 5	1	5	10	10	5	1				
Row 6	1	6	15	20	15	6	1			
Row 7	1	7	21	35	35	21	7	1		
Row 8	1	8	28	56	70	56	28	8	1	
Row 9	1	9	36	84	126	126	84	36	9	1

c. How many students can be elected?

There are 4 openings on the student council, so 4 students can be elected.

d. How many different combinations of 4 elected students can be created using the 9 candidates?

Find term 4 in row 9 of the triangle.

Remember that the first term is "term 0," so you are actually finding the fifth term.

There are 126 ways 4 students can be elected.

Name: **Date:**

A–SSE.1b★; A–SSE.2; A–APR.4 • Algebra

Polynomial Identities

Common Core State Standards

A–SSE.1

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b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.*

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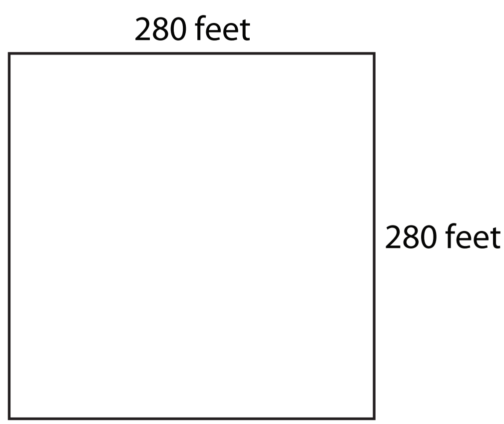
Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

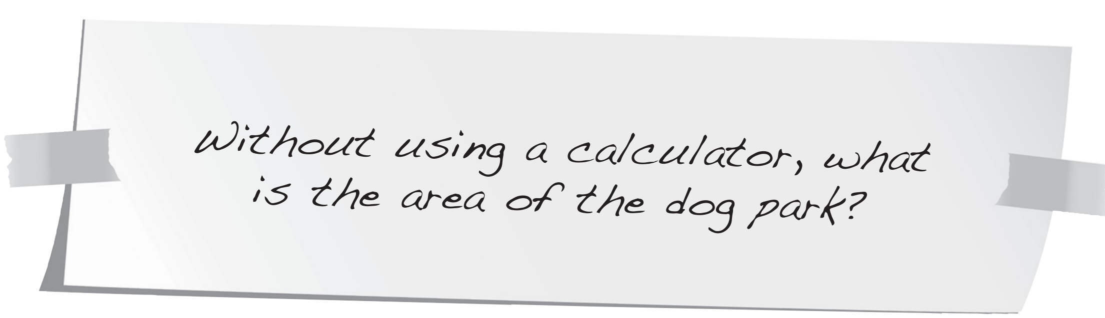
A–APR.4

Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*

Problem-Based Task: How Big Is the Dog Park?

City planners are designing a new dog park. The park will be a square, with the dimensions shown in the diagram below. The area of the park is the square of the side length: area = 280^2 feet^2. Without using a calculator, what is the area of the dog park? Use polynomial identities to support your answer.





Problem-Based Task: How Big Is the Dog Park?

Coaching

a. Which polynomial identities can be used to find the square of a number?

b. How can the side length of the park be rewritten as a sum or difference?

c. Using the sum or difference of the side length, and a polynomial identity, what is an equivalent expression that represents the area of the dog park?

d. Using the expression and the identity, what is the area of the park?

Problem-Based Task: How Big Is the Dog Park?

Coaching Sample Responses

a. Which polynomial identities can be used to find the square of a number?

There are three identities that can be used to find the square of a value:

- Square of Sums Identity (for two variables): $(a + b)^2 = a^2 + 2ab + b^2$
- Square of Differences Identity: $(a - b)^2 = a^2 - 2ab + b^2$
- Square of Sums Identity (for three variables): $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

b. How can the side length of the park be rewritten as a sum or difference?

Choose a sum or difference that includes two numbers that can be used to easily find products and squares.

One way to rewrite the side length, 280 feet, is to use the difference 300 – 20.

c. Using the sum or difference of the side length, and a polynomial identity, what is an equivalent expression that represents the area of the dog park?

The area of the park is the square of the side length, or 280^2.

If this number is rewritten using the difference 300 – 20, the area is $(300 - 20)^2$.

The Square of Differences Identity, $(a - b)^2 = a^2 - 2ab + b^2$, can be used to rewrite the expression for the area.

$(300 - 20)^2 = 300^2 - 2(300)(20) + 20^2$

d. Using the expression and the identity, what is the area of the park?

Evaluate each term in the expression.

$(300 - 20)^2 = 300^2 - 2(300)(20) + 20^2$

$= 90{,}000 - 2(300)(20) + 20^2$

$= 90{,}000 - 12{,}000 + 20^2$

$= 90{,}000 - 12{,}000 + 400$

$= 78{,}400$

The area of the park is 78,400 feet^2.

