**Texas Essential Knowledge and Skills** 

# Advanced Quantitative Reasoning



**Program Overview** 



This program was developed and reviewed by experienced math educators who have both academic and professional backgrounds in mathematics. This ensures: freedom from mathematical errors, grade level appropriateness, freedom from bias, and freedom from unnecessary language complexity.

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### **Table of Contents for Instructional Units**

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Lesson 1.2: Comparing Data Sets (TEKS.AQR.4P)

Lesson 1.3: Interpreting Data Sets (TEKS.AQR.4O, TEKS.AQR.4P)

#### **Conceptual Activities**

Desmos. "LEGO Prices." (TEKS.AQR.4P)

GeoGebra. "Lotto Tickets Simulation." (TEKS.AQR.4P)

GeoGebra. "Dot Plot Maker." (TEKS.AQR.4P)

GeoGebra. "Measures of Center." (TEKS.AQR.4P)

GeoGebra. "Visualize Measures of Center and Spread." (TEKS.AQR.4P)

GeoGebra. "Outlier Influence." (TEKS.AQR.4P)

#### **Conceptual Task**

Desmos. "Polygraph: Histograms." (TEKS.AQR.4P)

#### **Topic B: Using the Normal Curve**

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Lesson 1.5: Standard Normal Calculations (TEKS.AQR.4P)

Lesson 1.6: Assessing Normality (TEKS.AQR.4P)

#### **Conceptual Activities**

GeoGebra. "192 Normal Distribution Simulation: Bike & Wall." (TEKS.AQR.4P)

GeoGebra. "Algebra 2 Lesson 7." (TEKS.AQR.4P)

GeoGebra. "Normal Curve Demonstration." (TEKS.AQR.4P)

Illustrative Math. "Normal Distributions." (TEKS.AQR.4P)

#### **Conceptual Tasks**

Tons of Tuna, Parts 1 and 2 (TEKS.AQR.4P)

Illustrative Mathematics. "Should We Send Out a Certificate?" (TEKS.AQR.4P)

#### Topic C: Summarizing, Representing, and Finding Patterns in Data

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Lesson 1.8: Solving Problems Given Functions Fitted to Data (TEKS.AQR.3A)

Lesson 1.9: Analyzing Residuals (TEKS.AQR.3A)

Lesson 1.10: Fitting Linear Functions to Data (TEKS.AQR.3A)

#### **Conceptual Activity**

Desmos. "Predicting Movie Ticket Prices." (TEKS.AQR.3A)

#### **Conceptual Task**

Time to Print in 3D, Parts 1 and 2 (TEKS.AQR.3A)

#### Table of Contents for Instructional Units

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Lesson 1.13: Distinguishing Between Correlation and Causation (TEKS.AQR.3B)

#### **Conceptual Activities**

GeoGebra. "Correlation Experimentation." (TEKS.AQR.3A)

GeoGebra. "Correlation Sliders." (TEKS.AQR.3B)

#### **Conceptual Task**

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#### **Answer Keys**

**Unit 1 Assessment** 

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Lesson 2.3: Understanding Independent Events (TEKS.AQR.4C)

#### **Topic B: Conditional Probability**

Lesson 2.4: Introducing Conditional Probability (TEKS.AQR.4C, TEKS.AQR.4D)

Lesson 2.5: Using Two-Way Frequency Tables (TEKS.AQR.4A)

Lesson 2.6: The Multiplication Rule (TEKS.AQR.4C, TEKS.AQR.4D)

#### **Topic C: Combinatorics**

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Lesson 2.8: Probability with Combinatorics (TEKS.AQR.2E)

#### **Topic D: Decision Making with Probability**

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Lesson 2.10: Making Decisions Using Probability (TEKS.AQR.4E)

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Lesson 2.12: Expected Value (TEKS.AQR.4F)

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#### **Performance Task**

Playing Roulette (TEKS.AQR.4F)

#### **Answer Keys**

**Unit 2 Assessment** 

#### Table of Contents for Instructional Units

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Lesson 3.2: Simple Random Sampling (TEKS.AQR.4M, TEKS.AQR.4Q)

Lesson 3.3: Other Methods of Random Sampling (TEKS.AQR.4M, TEKS.AQR.4O, TEKS.AQR.4Q)

#### **Conceptual Activity**

GeoGebra. "Sampling from a population of ordered pairs."

#### Topic B: Surveys, Experiments, and Observational Studies

Lesson 3.4: Identifying Surveys, Experiments, and Observational Studies (TEKS.AQR.4L)

Lesson 3.5: Designing Surveys, Experiments, and Observational Studies (TEKS.AQR.4L)

#### **Conceptual Task**

Studying Shoppers, Parts 1 and 2 (TEKS.AQR.4L)

#### **Topic C: Estimating Sample Proportions and Sample Means**

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Lesson 3.7: The Binomial Distribution (TEKS.AQR.4I)

Lesson 3.8: Estimating Sample Means (TEKS.AQR.4I)

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#### **Topic D: Comparing Treatments and Reading Reports**

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Lesson 3.11: Designing and Simulating Treatments (TEKS.AQR.4H, TEKS.AQR.4J, TEKS.AQR.4K)

Lesson 3.12: Reading Reports (TEKS.AQR.2B, TEKS.AQR.4H, TEKS.AQR.4J, TEKS.AQR.4K)

#### **Topic E: Making and Analyzing Decisions**

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TEKS.AQR.4T)

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#### **Answer Keys**

**Unit 3 Assessment** 

#### Table of Contents for Instructional Units

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Lesson 4.3: Rewriting Exponential Expressions and Equations (TEKS.AQR.3C)

Lesson 4.4: Building Functions Including Parameters (TEKS.AQR.3C)

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Lesson 4.6: Common Logarithms (TEKS.AQR.3C)

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Lesson 4.11: Using Trigonometric Functions to Model Periodic Phenomena (TEKS.AQR.3D)

#### **Topic E: Modeling Trigonometric Functions**

Lesson 4.12: Graphing the Sine Function (TEKS.AQR.3D)

Lesson 4.13: Graphing the Cosine Function (TEKS.AQR.3D)

#### **Topic F: Choosing a Model**

Lesson 4.14: Linear, Exponential, and Quadratic Functions (TEKS.AQR.3C)

Lesson 4.15: Piecewise, Step, and Absolute Value Functions (TEKS.AQR.3E)

#### **Topic G: Algorithms**

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Lesson 4.17: Creating and Applying Algorithms (TEKS.AQR.2H)

#### **Answer Keys**

**Unit 4 Assessment** 

#### Table of Contents for Instructional Units

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#### **Unit 5 Pre-Assessment**

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Lesson 5.2: Using Operations on Matrices (TEKS.AQR.2F)

#### **Topic B: Vectors**

Lesson 5.3: Representing and Modeling with Vector Quantities (TEKS.AQR.2F)

Lesson 5.4: Performing Operations on Vectors (TEKS.AQR.2F)

Lesson 5.5: Determinants and Vectors (TEKS.AQR.2F)

#### **Topic C: Using Matrices to Solve Systems of Equations**

Lesson 5.6: Representing a System of Linear Equations as a Single Matrix (TEKS.AQR.2F)

Lesson 5.7: Finding the Inverse of a Matrix and Using It to Solve

a System of Equations (TEKS.AQR.2F)

#### **Performance Tasks**

Computer Animation with Matrices (TEKS.AQR.2F)

Problem Solving with Matrices (TEKS.AQR.2F)

#### **Answer Keys**

**Unit 5 Assessment** 

#### **Unit 6: Finance**

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Lesson 6.1: Creating Equations and Inequalities—Gross Pay (TEKS.AQR.3F)

Lesson 6.2: Creating Equations in Context—Net Pay (TEKS.AQR.3F)

Lesson 6.3: Income and Constraints (TEKS.AQR.3F)

#### **Topic B: Credit**

Lesson 6.4: Solving Linear Equations—Simple Interest (TEKS.AQR.3G)

Lesson 6.5: Analyzing Credit Offers with Linear and

Exponential Equations (TEKS.AQR.3G)

#### **Topic C: Loans and Financed Purchases**

Lesson 6.6: Recursion and Sequences—Payment Plans (TEKS.AQR.3H)

Lesson 6.7: Finite Geometric Series—Amortized Loans (TEKS.AQR.3H)

#### Table of Contents for Instructional Units

#### **Topic D: Banking**

Lesson 6.8: Interpreting Complicated Expressions—Bank Statements and Savings Accounts (TEKS.AQR.3H)

Lesson 6.9: Analyzing Savings Account Options Using Equations and Inequalities (TEKS.AQR.3H)

#### **Topic E: Investing**

Lesson 6.10: Interpreting Expressions and Equations—Stocks and Shares (TEKS.AQR.3H)

Lesson 6.11: Interpreting Stock Parameters (TEKS.AQR.3H)

Lesson 6.12: Reading Stock Reports (TEKS.AQR.3H)

#### **Performance Task**

Automobile Ownership (TEKS.AQR.3H)

#### **Answer Keys**

**Unit 6 Assessment** 

#### **Unit 7: Geometry**

#### **Unit 7 Pre-Assessment**

#### **Topic A: Investigating Properties of Dilations**

Lesson 7.1: Investigating Properties of Parallelism and the Center (TEKS.AQR.2C)

Lesson 7.2: Investigating Scale Factors (TEKS.AQR.2C)

#### **Topic B: Defining and Applying Similarity**

Lesson 7.3: Defining Similarity (TEKS.AQR.2D)

Lesson 7.4: Applying Similarity Using the Angle-Angle (AA) Criterion (TEKS.AQR.2D)

#### **Conceptual Task**

Similarity Investigation (TEKS.AQR.2D)

#### **Topic C: Proving Similarity**

Lesson 7.5: Proving Triangle Similarity Using Side-Angle-Side (SAS) and Side-Side-Side (SSS) Similarity (TEKS.AQR.2D)

Lesson 7.6: Working with Ratio Segments (TEKS.AQR.2D)

Lesson 7.7: Proving the Pythagorean Theorem Using Similarity (TEKS.AQR.2D)

Lesson 7.8: Solving Problems Using Similarity and Congruence (TEKS.AQR.2D)

Lesson 7.9: Special Right Triangles (TEKS.AQR.2D)

#### **Topic D: Exploring Trigonometric Ratios**

Lesson 7.10: Defining Trigonometric Ratios (TEKS.AQR.2D)

Lesson 7.11: Exploring Sine and Cosine As Complements (TEKS.AQR.2D)

#### **Table of Contents for Instructional Units**

#### **Topic E: Applying Trigonometric Ratios**

Lesson 7.12: Calculating Sine, Cosine, and Tangent (TEKS.AQR.2D)

Lesson 7.13: Problem Solving with the Pythagorean Theorem and Trigonometry (TEKS.AQR.2D)

#### **Conceptual Tasks**

High Altitude Trigonometry (TEKS.AQR.2D)

Triangles? Yeah, Right. (TEKS.AQR.2D)

#### **Topic F: Trigonometry of General Angles**

Lesson 7.14: Proving the Law of Sines (TEKS.AQR.2D)

Lesson 7.15: Proving the Law of Cosines (TEKS.AQR.2D)

Lesson 7.16: Applying the Laws of Sines and Cosines (TEKS.AQR.2D)

#### **Conceptual Task**

Tricky Triangles (TEKS.AQR.2D)

#### **Topic G: Using Truth Tables**

Lesson 7.17: Using Truth Tables (TEKS.AQR.4G)

#### **Answer Key**

**Unit 7 Assessment** 

### Introduction to the Program

#### Introduction

The *TEKS Advanced Quantitative Reasoning Program* is a complete set of materials developed to be aligned to the Texas Essential Knowledge and Skills (TEKS) and the Advanced Quantitative Reasoning content map. Topics are built around accessible core curricula, ensuring that the *TEKS Advanced Quantitative Reasoning Program* is useful for college-ready students and diverse classrooms.

This program realizes the benefits of exploratory and investigative learning and employs a variety of instructional models to meet the learning needs of students with a range of abilities.

The *TEKS Advanced Quantitative Reasoning Program* includes components that support problem-based learning, instruct and coach as needed, provide practice, and assess students' skills. Instructional tools and strategies are embedded throughout.

#### The program includes:

- More than 165 hours of lessons
- Essential Questions for each instructional topic
- Vocabulary
- Instruction and Guided Practice
- Sets of standards-based Scaffolded Practice and Practice problems
- Problem-based Tasks and Coaching questions
- Step-by-step graphing calculator instructions for the TI-Nspire and the TI-83/84
- Performance Tasks to promote collaborative learning and problem-solving skills
- Aligned open education resources that enhance procedural fluency and conceptual understanding
- Embedded Instructional Strategies to enable access for all students

#### **Purpose of Materials**

The *TEKS Advanced Quantitative Reasoning Program* has been organized to coordinate with the TEKS Advanced Quantitative Reasoning content map and specifications from the Texas Essential Knowledge and Skills.

Each topic includes activities that offer opportunities for exploration and investigation. These activities incorporate concept and skill development and guided practice, then move on to the application of new skills and concepts in problem-solving situations. Throughout the lessons and activities, problems are contextualized to enhance rigor and relevance.

#### Introduction to the Program

This program includes all the topics addressed in the TEKS Advanced Quantitative Reasoning content map. These include:

- Descriptive Statistics
- Probability
- Inferences and Conclusions from Data
- Modeling with Functions
- Matrices
- Finance
- Geometry

The Mathematical Process Standards described in the Texas Essential Knowledge and Skills are infused throughout. The student is expected to:

- (A) apply mathematics to problems arising in everyday life, society, and the workplace;
- (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
- (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
- (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
- (E) create and use representations to organize, record, and communicate mathematical ideas;
- (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
- (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

#### Structure of the Teacher Resource

The *TEKS Advanced Quantitative Reasoning Teacher Resource* materials are completely reproducible. The Program Overview is the first section. This section helps you to navigate the materials, offers a collection of research-based Instructional Strategies along with their literacy connections and implementation suggestions, and shows the correlation between the Texas Essential Knowledge and Skills and the Advanced Quantitative Reasoning content map and course requirements.

#### Introduction to the Program

The remaining materials focus on building math content knowledge and conceptual understanding through application of the units in the *Advanced Quantitative Reasoning* program: Descriptive Statistics, Probability, Inferences and Conclusions from Data, Modeling with Functions, Matrices, Finance, and Geometry. The units in the *TEKS Advanced Quantitative Reasoning Program* are designed to be flexible so that you can mix and match activities as the needs of your students and your instructional style dictate.

The Performance Tasks correspond to the content in selected units and provide students with the opportunity to apply concepts and skills, while you have a chance to circulate, observe, speak to individuals and small groups, and informally assess and plan.

Each unit includes a pre-assessment and unit assessment, and each topic ends with a progress assessment. These allow you to assess students' progress as you move from topic to topic, enabling you to gauge how well students have understood the material and to differentiate as appropriate.

#### Glossary

The Glossary contains vocabulary terms and formulas from throughout the program, organized alphabetically by units. Each listing provides the term and the definition in both English and Spanish.

### **Unit Structure**

All of the instructional units have common features. Each unit begins with a list of all the standards addressed in the topics and a list of one or more conceptual activities. Each unit also begins with a pre-assessment. Each topic begins with an overview of the standards addressed in the topic; Essential Questions; vocabulary (titled "Words to Know"); and a list of recommended websites to be used as additional resources.

Each lesson begins with a list of identified prerequisite skills that students need to have mastered in order to be successful with the new material in the upcoming lesson. This is followed by an introduction, key concepts, common errors/misconceptions, scaffolded practice problems, guided practice examples, a problem-based task with coaching questions and sample responses, a closure activity, and practice. Each topic ends with a progress assessment to evaluate students' learning.

All of the components are described below and on the following pages for your reference.

#### **Standards Correlations**

In this section, you'll find a comprehensive list of the Texas Essential Knowledge and Skills addressed in each lesson.

#### **Conceptual Activities**

Conceptual understanding serves as the foundation on which to build deeper understanding of mathematics. In an effort to build conceptual understanding of mathematical ideas and to provide more than procedural fluency and application, links to interactive open education and Desmos resources are included. (*Note*: These website links will be monitored and repaired or replaced as necessary.) These and many other open educational resources (OERs) are also accessible through the Learning Object Repository as separate objects that can be assigned to students.

#### **Unit Pre-Assessment and Answer Key with TEKS**

This can be used to gauge students' prior knowledge and to inform instructional planning. The assessment is followed by an answer key that lists the Texas Essential Knowledge and Skills addressed by each problem.

#### **Instructional Strategies**

These research-based strategies are intended to provide additional support for teachers using Walch's resources. Implementation guides for these instructional strategies can be found later in the program overview, while icons are located throughout the instructional resources to indicate appropriate strategies at the point of use. Inclusion of these strategies along with additional resources for English as a Second Language (ESL) students, SWD students, and struggling readers will provide teachers with a wide range of instructional support.

#### **Unit Structure**

#### **Texas Essential Knowledge and Skills for the Topic**

All standards that are addressed in the entire topic are listed.

#### **Essential Questions**

These are intended to guide students' thinking as they proceed through the lesson. By the end of each lesson, students should be able to respond to the questions.

#### **Words to Know**

A list of vocabulary terms that appear in the unit are provided as background information for instruction or to review key concepts that are addressed in the lesson. Each term is followed by a numerical reference to the first lesson in which the term is defined.

#### **Recommended Resources**

This is a list of websites that can be used as additional resources. Some websites are games; others provide additional examples and/or explanations. The links for these resources are live in the PDF version of the Teacher Resource. (*Note*: These website links will be monitored and repaired or replaced as necessary.) Each Recommended Resource is also accessible through Walch's cloud-based Curriculum Engine Learning Object Repository as a separate learning object that can be assigned to students.

#### Warm-Up

Each warm-up takes approximately 5 minutes and addresses either prerequisite and critical-thinking skills or previously taught math concepts.

#### Warm-Up Debrief

Each debrief provides the answers to the warm-up questions, and offers suggestions for situations in which students might have difficulties. A section titled Connection to the Lesson is also included in the debrief to help answer students' questions about the relevance of the particular warm-up activity to the upcoming instruction. Warm-Ups with debriefs are also provided in PowerPoint presentations.

#### **Identified Prerequisite Skills**

This list cites the skills necessary to be successful with the new material.

#### Introduction

This brief section gives a description of the concepts about to be presented and often contains some Words to Know.

#### **Unit Structure**

#### **Key Concepts**

Provided in bulleted form, this instruction highlights the important ideas and/or processes for meeting the standard.

#### **Graphing Calculator Directions**

Step-by-step instructions for using a TI-Nspire and a TI-83/84 are provided whenever graphing calculators are referenced.

#### **Common Errors/Misconceptions**

This is a list of the common errors students make when applying Key Concepts. The list suggests what to watch for when students arrive at an incorrect answer or are struggling with solving the problems.

#### **Scaffolded Practice (Printable Practice)**

This set of 10 printable practice problems provides introductory level skill practice for the lesson. This practice set can be used during instruction time.

#### **Guided Practice**

This section provides step-by-step examples of applying the Key Concepts. The three to five examples are intended to aid during initial instruction, but are also for individuals needing additional instruction and/or for use during review and test preparation.

#### **Enhanced Instructional PowerPoint (Presentation)**

Each lesson includes an instructional PowerPoint presentation with the following components: Warm-Up, Key Concepts, and Guided Practice. Selected Guided Practice examples include GeoGebra applets. These instructional PowerPoints are downloadable and editable.

#### **Problem-Based Task**

This activity can serve as the centerpiece of a problem-based lesson, or it can be used to walk students through the application of the standard, prior to traditional instruction or at the end of instruction. The task makes use of critical-thinking skills.

#### **Optional Problem-Based Task Coaching Questions with Sample Responses**

These questions scaffold the task and guide students to solving the problem(s) presented in the task. They should be used at the discretion of the teacher for students requiring additional support. The Coaching Questions are followed by answers and suggested appropriate responses to the coaching questions. In some cases answers may vary, but a sample answer is given for each question.

#### **Unit Structure**

#### **Recommended Closure Activity**

Students are given the opportunity to synthesize and reflect on the lesson through a journal entry or discussion of one or more of the Essential Questions.

#### Printable Practice (Sets A and B) and Interactive Practice (Set A)

Each lesson includes two sets of practice problems to support students' achievement of the learning objectives. They can be used in any combination of teacher-led instruction, cooperative learning, or independent application of knowledge. Each Practice A is also available as an interactive Learnosity activity with Technology-Enhanced Items.

#### **Conceptual Tasks**

These engaging tasks provide opportunities for students to deepen their understanding and develop their conceptual knowledge of math concepts. These tasks provide multiple entry points and are accessible for ALL learners.

#### **Topic Progress Assessment and Answer Key with TEKS**

Each topic ends with 10 multiple-choice questions, as well as one extended-response question that incorporates critical thinking and writing components. This can be used to document the extent to which students grasp the concepts and skills addressed during instruction. The answer key that follows correlates each problem to the targeted Standard(s).

#### **Answer Key**

Answers for all of the Warm-Ups and practice problems are provided following each unit.

#### **Unit Assessment and Answer Key with TEKS**

Each unit ends with 12 multiple-choice questions and three extended-response questions that incorporate critical thinking and writing components. This can be used to document the extent to which students grasped the concepts and skills of each unit. Targeted TEKS are listed in the answer key.

#### **Performance Task**

Selected units include an extension activity to provide students with opportunities to practice, reinforce, and apply mathematical skills and concepts to a real-world task.

## **Standards Correlations**

Each topic in this TEKS Advanced Quantitative Reasoning program was written specifically to address the Texas Essential Knowledge and Skills (TEKS). Each topic lists the standards covered in all the lessons, and each lesson lists the standards addressed in that particular lesson. In this section, you'll find a comprehensive list mapping the lessons to the TEKS.

		<b>Unit 1: Descriptive Statistics</b>	
Topic	Lesson number	Title	Standard(s)
Topic A	Representing	g and Interpreting Data	
	1.1	Representing Data Sets	TEKS.AQR.4P
	1.2	Comparing Data Sets	TEKS.AQR.4P
	1.3	Interpreting Data Sets	TEKS.AQR.4O TEKS.AQR.4P
Topic B	Using the No	rmal Curve	
_	1.4	Normal Distributions and the 68–95–99.7 Rule	TEKS.AQR.4P
	1.5	Standard Normal Calculations	TEKS.AQR.4P
	1.6	Assessing Normality	TEKS.AQR.4P
Topic C	Summarizing, Representing, and Finding Patterns in Data		
	1.7	Summarizing Data Using Two-Way Frequency Tables	TEKS.AQR.3A
	1.8	Solving Problems Given Functions Fitted to Data	TEKS.AQR.3A
	1.9	Analyzing Residuals	TEKS.AQR.3A
	1.10	Fitting Linear Functions to Data	TEKS.AQR.3A
Topic D	Interpreting	Statistical Models	
	1.11	Interpreting Key Features	TEKS.AQR.3A
	1.12	Calculating and Interpreting the Correlation Coefficient	TEKS.AQR.3A
	1.13	Distinguishing Between Correlation and Causation	TEKS.AQR.3B

		Unit 2: Probability	
Topic	Lesson number	Title	Standard(s)
Topic A	Events		
	2.1	Describing Events	TEKS.AQR.4B
	2.2	The Addition Rule	TEKS.AQR.4B
	2.3	Understanding Independent Events	TEKS.AQR.4C
Topic B	<b>Conditional</b>	Probability	
	2.4	Introducing Conditional Probability	TEKS.AQR.4C TEKS.AQR.4D
	2.5	Using Two-Way Frequency Tables	TEKS.AQR.4A
	2.6	The Multiplication Rule	TEKS.AQR.4C TEKS.AQR.4D
Topic C	Combinatori	cs	
	2.7	Combinations and Permutations	TEKS.AQR.2E
	2.8	Probability with Combinatorics	TEKS.AQR.2E
Topic D	<b>Decision Ma</b>	king with Probability	
	2.9	Determining Fairness	TEKS.AQR.4E
	2.10	Making Decisions Using Probability	TEKS.AQR.4E
Topic E	<b>Probability I</b>	Distributions	
	2.11	Creating Graphs of Probability Distributions	TEKS.AQR.4F
	2.12	Expected Value	TEKS.AQR.4F
	2.13	Developing Probability Distributions	TEKS.AQR.4F
	2.14	Using Probability Distributions to Evaluate Outcomes	TEKS.AQR.4F

	Uni	t 3: Inferences and Conclusions from Data	
Topic	Lesson number	Title	Standard(s)
Topic A	Populations	<b>Versus Random Samples and Random Sampling</b>	g
	3.1	Differences Between Populations and Samples	TEKS.AQR.4M TEKS.AQR.4Q
	3.2	Simple Random Sampling	TEKS.AQR.4M TEKS.AQR.4Q
	3.3	Other Methods of Random Sampling	TEKS.AQR.4M TEKS.AQR.4O TEKS.AQR.4Q
Topic B	Surveys, Exp	periments, and Observational Studies	
	3.4	Identifying Surveys, Experiments, and Observational Studies	TEKS.AQR.4L
	3.5	Designing Surveys, Experiments, and Observational Studies	TEKS.AQR.4L
Topic C	Estimating S	Sample Proportions and Sample Means	
	3.6	Estimating Sample Proportions	TEKS.AQR.4I
	3.7	The Binomial Distribution	TEKS.AQR.4I
	3.8	Estimating Sample Means	TEKS.AQR.4I
	3.9	Estimating with Confidence	TEKS.AQR.4I TEKS.AQR.4Q
Topic D	Comparing T	Freatments and Reading Reports	
	3.10	Evaluating Treatments	TEKS.AQR.4H TEKS.AQR.4J TEKS.AQR.4K
	3.11	Designing and Simulating Treatments	TEKS.AQR.4H TEKS.AQR.4J TEKS.AQR.4K
	3.12	Reading Reports	TEKS.AQR.2B TEKS.AQR.4H TEKS.AQR.4J TEKS.AQR.4K

Topic	Lesson number	Title	Standard(s)
Topic E	Making and	Analyzing Decisions	
			TEKS.AQR.2G
	3.13	Making Decisions	TEKS.AQR.4R
	3.13		TEKS.AQR.4S
			TEKS.AQR.4T
			TEKS.AQR.2G
	3.14 Analyzing Decisions	TEKS.AQR.4R	
		TEKS.AQR.4S	
			TEKS.AQR.4T

		<b>Unit 4: Modeling with Functions</b>	
Topic	Lesson number	Title	Standard(s)
Topic A	Units of Measure		
	4.1	Converting Units	TEKS.AQR.2A
	4.2	Modeling with Units and Precision in Modeling	TEKS.AQR.2A
Topic B	<b>Modeling Ex</b>	ponential Functions	
	4.3	Rewriting Exponential Expressions and Equations	TEKS.AQR.3C
	4.4	Building Functions Including Parameters	TEKS.AQR.3C
Topic C	<b>Modeling Lo</b>	garithmic Functions	
	4.5	Logarithmic Functions as Inverses	TEKS.AQR.3C
	4.6	Common Logarithms	TEKS.AQR.3C
	4.7	Natural Logarithms	TEKS.AQR.3C
	4.8	Graphing Logarithmic Functions	TEKS.AQR.3C
	4.9	Interpreting Logarithmic Models	TEKS.AQR.3C
Topic D	<b>Graphs of Ti</b>	igonometric Functions	
	4.10	Periodic Phenomena and Amplitude, Frequency, and Midline	TEKS.AQR.3D
	4.11	Using Trigonometric Functions to Model Periodic Phenomena	TEKS.AQR.3D
Topic E	<b>Modeling Tr</b>	igonometric Functions	
	4.12	Graphing the Sine Function	TEKS.AQR.3D
	4.13	Graphing the Cosine Function	TEKS.AQR.3D

Topic	Lesson number	Title	Standard(s)	
<b>Topic F</b>	<b>Choosing a M</b>	<b>fodel</b>		
	4.14	Linear, Exponential, and Quadratic Functions	TEKS.AQR.3C	
	4.15	Piecewise, Step, and Absolute Value Functions	TEKS.AQR.3E	
Topic G	Algorithms			
	4.16	Introduction to Algorithms	TEKS.AQR.2H	
	4.17	Creating and Applying Algorithms	TEKS.AQR.2H	

		<b>Unit 5: Matrices</b>	
Topic	Lesson number	Title	Standard(s)
Topic A	Manipulatin	g Matrices	
	5.1	Performing Operations on Matrices	TEKS.AQR.2F
	5.2	Using Operations on Matrices	TEKS.AQR.2F
Topic B			
	5.3	Representing and Modeling with Vector Quantities	TEKS.AQR.2F
	5.4	Performing Operations on Vectors	TEKS.AQR.2F
	5.5	Determinants and Vectors	TEKS.AQR.2F
Topic C	<b>Using Matric</b>	es to Solve Systems of Equations	
	5.6	Representing a System of Linear Equations as a Single Matrix	TEKS.AQR.2F
	5.7	Finding the Inverse of a Matrix and Using It to Solve a System of Equations	TEKS.AQR.2F

		Unit 6: Finance	
Topic	Lesson number	Title	Standard(s)
Topic A	Income		
	6.1	Creating Equations and Inequalities—Gross Pay	TEKS.AQR.3F
	6.2	Creating Equations in Context—Net Pay	TEKS.AQR.3F
	6.3	Income and Constraints	TEKS.AQR.3F
Topic B	Credit		
	6.4	Solving Linear Equations—Simple Interest	TEKS.AQR.3G
	6.5	Analyzing Credit Offers with Linear and Exponential Equations	TEKS.AQR.3G
Topic C			
	6.6	Recursion and Sequences—Payment Plans	TEKS.AQR.3H
	6.7	Finite Geometric Series-—Amortized Loans	TEKS.AQR.3H
Topic D	Banking		
	6.8	Interpreting Complicated Expressions—Bank Statements and Savings Accounts	TEKS.AQR.3H
	6.9	Analyzing Savings Account Options Using Equations and Inequalities	TEKS.AQR.3H
Topic E	Investing		
	6.10	Interpreting Expressions and Equations—Stocks and Shares	TEKS.AQR.3H
	6.11	Interpreting Stock Parameters	TEKS.AQR.3H
	6.12	Reading Stock Reports	TEKS.AQR.3H

Unit 7: Geometry			
Topic	Lesson number	Title	Standard(s)
Topic A	Investigating	g Properties of Dilations	
	7.1	Investigating Properties of Parallelism and the Center	TEKS.AQR.2C
	7.2	Investigating Scale Factors	TEKS.AQR.2C
Topic B	Defining and Applying Similarity		
	7.3	Defining Similarity	TEKS.AQR.2D
	7.4	Applying Similarity Using the Angle-Angle (AA) Criterion	TEKS.AQR.2D
Topic C	Proving Similarity		
	7.5	Proving Triangle Similarity Using Side-Angle-Side (SAS) and Side-Side (SSS) Similarity	TEKS.AQR.2D
	7.6	Working with Ratio Segments	TEKS.AQR.2D
	7.7	Proving the Pythagorean Theorem Using Similarity	TEKS.AQR.2D
	7.8	Solving Problems Using Similarity and Congruence	TEKS.AQR.2D
	7.9	Special Right Triangles	TEKS.AQR.2D
Topic D	Exploring Trigonometric Ratios		
	7.10	Defining Trigonometric Ratios	TEKS.AQR.2D
	7.11	Exploring Sine and Cosine As Complements	TEKS.AQR.2D
Topic E	Applying Trigonometric Ratios		
	7.12	Calculating Sine, Cosine, and Tangent	TEKS.AQR.2D
	7.13	Problem Solving with the Pythagorean Theorem and Trigonometry	TEKS.AQR.2D
Topic F	Trigonometry of General Angles		
	7.14	Proving the Law of Sines	TEKS.AQR.2D
	7.15	Proving the Law of Cosines	TEKS.AQR.2D
	7.16	Applying the Laws of Sines and Cosines	TEKS.AQR.2D
Topic G	Using Truth Tables		
	7.17	Using Truth Tables	TEKS.AQR.4G