EMPower Plus
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Everyday Number Sense

Mental Math and Visual Models

TEACHER BOOK

Mathematical Concepts Covered for Everyday Number Sense: Mental Math and Visual Models

Book Description: Students solve problems with whole numbers using mental math strategies with benchmarks of 10, 100, and 1000. Number lines, arrays, and diagrams support conceptual understanding of number relationships and the four operations.

Lesson Number:	Lesson Name:	Mathematical Concepts/Topics Covered
Opening the Unit	Everyday Number Sense	• Personal math experiences • Mental math skills • Fluency with visual models and symbolic expressions • Number properties such as the commutative and distributive properties
Lesson 1	Close Enough with Mental Math	• Mental math strategies • The role of commutativity in addition of whole numbers
Lesson 2	Mental Math in the Checkout Line	• Totals computed mentally by rounding and then adjusting • Mental math processes notated mathematically • Generalizations about equations
Lesson 3	Traveling with Numbers	• Numbers on a number line located, put in order, and operated on • Numbers rounded to the nearest 10 and 100
Lesson 4	Traveling in Time	• Mental math strategies explained with a number line • Mental math and number-line actions recorded with equations • Counting by 10's and 1's to solve addition and subtraction problems • Generalizations about how addition and subtraction behave in equations
Lesson 5	Meanings and Methods for Subtraction	• Three models (or interpretations) for subtraction identified and used • Algorithms for addition and subtraction examined; why they work
Lesson 6	Extending the Line	• Negative and positive numbers located on a number line • Difference between two numbers determined
Lesson 7	Ups and Downs with Addition	• Visual models and symbolic notation to express addition with integers • Patterns identified that occur when adding integers (e.g., commutative and associative properties)

Lesson 8	Taking Your Winnings	• Composition of numbers in terms of 10's, 100's, and 1,000's examined and identified • Parentheses in expanded notation • Multiples of 10, 100, and 1,000 added and subtracted mentally • Mental calculations checked with calculators
Lesson 9	Patterns and Order	• Patterns for multiplying and dividing by 10, 100, 1,000 identified • Order of operations
Lesson 10	Picture this	• Connections between arrangements of objects in groups and arrays and written expressions • Equivalent expressions
Lesson 11	What's the Story?	• Situations represented with pictures and mathematical equations • Problem-solving strategies illustrated with equations and pictures • Squares and square roots of perfect squares • Multiplication with exponents
Lesson 12	Deal Me In	• Verbal language and symbolic notation for division matched to a concrete model • Mental math strategies for division applied to situations calling for splitting dollar amounts over time periods
Lesson 13	String It Along	• Direct measurements and scale used to find number of groups of a given size in a total • Mathematical symbols to express the action of division • Division related to multiplication and factors
Lesson 14	Making Do	• Units and precision for remainders • Remainders written and understood as decimals, fractions, and whole numbers • Paper-and-pencil division algorithms examined for why they work • Relationships among the four operations
Closing the Unit	Computer Lab	• Synthesis of the content • Areas of strength and weakness assessed

FACILITATING LESSON **10**

Picture This

How many are there? How do you know?

Synopsis

Students explore ways to visualize and represent numbers and operations with arrangements of objects in arrays (rows and columns) and equal groups. This lesson is the first of two lessons that focus on totaling and breaking apart groups of items while making connections among visual representations, mathematical expressions and equations, and word problems.

1. The class discusses ways to total items without individually counting each object. They record ideas using words and equations and by marking off arrays.
2. Individuals find at least two ways to record the total number of items in pictured groups, and then student pairs match equations with arrays.
3. Student pairs arrange collections of objects for ease in counting and write equations that reflect their arrangements.
4. Student pairs solve the *Garden Pathway* problem, and the whole class reviews the problem.
5. Students compare the algorithms they use to solve multiplication problems.
6. Using arrays, students make visible the subtotals resulting from the U.S. standard algorithm. Students see the distributive property at work.
7. The class summarizes what they learned about showing equations and writing equations. Then they discuss using math to find totals in their daily lives.

Objectives

- Write math expressions to reflect regular arrangements of objects in groups and arrays
- Represent equations using arrays and/or equal groups arranged to correspond with numbers and operations
- Identify **equivalent expressions**

Materials/Prep

- Colored markers
- Easel paper
- Uniform objects for counting, such as paper clips, pennies or counting chips, enough so that each student or pair can have between 30 and 60

Enlarge or project a copy of *Blackline Master 13: Windows.*

Take note of arrays around your classroom, for example, the arrangement of floor or ceiling tiles, desks, or windowpanes. Bring to class some items that come in arrays, for instance, egg cartons, six-pack containers, flats of garden plants, or booklets of stamps. These may be empty, as long as it is clear how many items they would contain. You could also take photographs and/or cut out magazine pictures showing things in rows and columns.

Opening Discussion

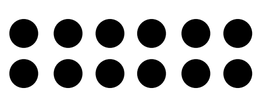
Set up the lesson by saying:

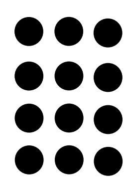
We have discussed different ways we calculate with numbers—mentally, with a number line, or with a calculator. Today we focus on a way of using math to describe an arrangement of objects. In this lesson, you will concentrate on visualizing the operations of multiplication and addition or subtraction.

Post and say aloud the definition for **arrays**:

Arrays show items arranged in rows and columns.

Show or sketch actual arrangements of 2 × 6 and 4 × 3 arrays.





Direct students to look around the class for examples of other arrays. Ask:

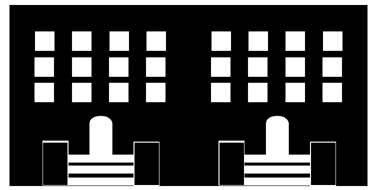
How many items are in this array? In this one?

How did you count them?

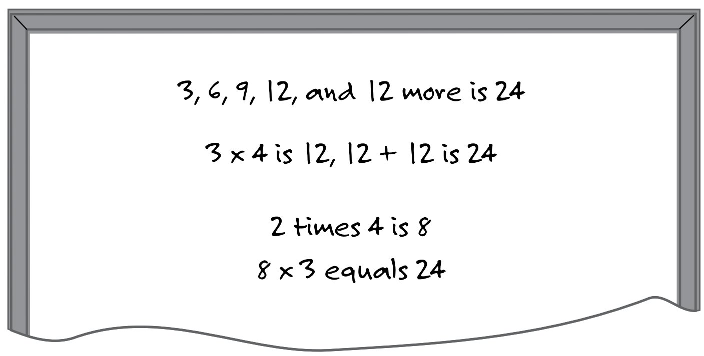
There are many ways to count. Students will likely say, "I counted one by one"; "I counted one row and then multiplied by the number of rows"; or "I skip counted by 4's." Ask whether they might have counted the number differently if they looked at the array differently, vertically instead of horizontally, for instance.

Display *Blackline Master 13: Windows* and say:

I know someone who *thinks* in arrays! She washes windows for a living and charges $5 a window. She needs to make quick estimates for her customers, so she never counts one by one. When she looks at a building like this, how many windows does she see?



Give students a moment to talk to a partner. As volunteers share approaches, record their ideas with equations. If someone says, "I counted by 3's," ask for details—what he or she said or thought when counting each group. Demonstrate on the board how the person saw the numbers in the array, and record the numbers as well as any explanation of mental arithmetic.



If someone explains, "I just knew that three 4's equal 12, so I added 12 and 12," post a copy of the picture and record:

- 3 × 4 = 12 (Box the first set of windows and label the sides with the numbers 3 and 4.)
- 12 + 12 = 24 (Box the second set of windows and label it "12.")

If the question "I saw 4 × 3—is that the same?" arises, take the opportunity to rotate the array pictured and ask, "Has the total changed?" You may wish to try a few more array examples to emphasize that 6 × 2 = 2 × 6, and these are equivalent because their total is equal.

Encourage a variety of approaches by asking questions such as these:

Did anyone count by 4's?

Did anyone use only addition? What did you do?

Did anyone use only multiplication? What did you do?

Did anyone multiply different numbers together or multiply numbers in a different order?

Did anyone use addition *and* multiplication?

As you record equations, ask whether anyone is familiar with other ways to record. For instance, if you record "3 × 4 = 12," ask whether anyone knows another way to show multiplication. Occasionally, vary your own recording to help students learn the different notation, in particular, use the raised dot and parentheses, as in 3 • 4 and 3(4).

Activity 1: Pictures and Numbers

Refer students to *Activity 1: Pictures and Numbers* (*Student Book*, p. 152).

Part 1

Students find totals without counting each individual item; instead they focus on groups of things.

They record at least two ways to find the total and mark the chosen pictures with different colored markers to show how they saw the numbers. Each student does this for two sets of objects.

Students who finish early can try the other *Part 1* items or find a partner and proceed to *Part 2*.

To review, summarize each problem separately and ask students to demonstrate how they saw the numbers.

In the faces problem, I noticed you counted in different ways and wrote different equations. Some people counted by 1's, some people added 10's, and others multiplied 10 by 10.

How are those ways alike? How are they different?

Is the same true for the chocolates problem, where some people added and multiplied and some people counted?

As connections are made between the expressions, take this opportunity to define **equivalent expressions—**those that result in the same total. Students may add this definition their list, *Student Book, Vocabulary*, p. 255.

Then focus on the counting:

Each way you counted involves multiplying some number a certain number of times.

With counting by 3's, 6's, 10's, etc., and repeated addition, you say or think, for instance, of all the 3's up to the total of 18. Multiplication is shorthand for that.

Part 2

Continue with *Part 2*, where the pictures are grids of squares and less literal.

First take suggestions for mathematical expressions, writing students' offerings on the board.

Probe:

Where did you see the 13 multiplied by 10? Where did you see the 13 multiplied by 1?

How does seeing the problem that way help you find the answer to the multiplication problem 13 × 11?

Have volunteers show the rest of the class how they matched the remaining expressions and arrays.

Bring into the discussion ideas about pulling apart numbers to make multiplying easier and "seeing" expressions in different formats that can be connected visually as well as by notation.

When you discuss the equation 10 × 10 + 13 × 1 + 3 × 10, highlight the 10 × 10 block, and introduce the raised **exponent** as another shorthand way to record an instance when a number is multiplied by itself (10^2). Numbers that can be drawn in arrays where the rows and columns are equal are called "**square numbers**." In this case, the exponent 2 tells you that two 10's are multiplied by one another. Ask:

Why do you think they are called "square numbers"?

What other square numbers can you think of? How would you write equations for them using an exponent? How would you write equations for them *without* using an exponent?

Activity 2: Counting Smart

Refer students to *Activity 2: Counting Smart* (*Student Book*, p. 156) and review the directions for *Part 1*. Give each pair a small handful of paper clips, pennies, or chips (about 30 to 60).

As students work, ask them how they are deciding on their arrangements and supporting the arrangements by recording the expressions. Ask students who are done quickly to create an arrangement that involves two operations.

Student pairs that finish early look at another pair's arrangement and explain with an equation how they found the total without counting each item. Pairs then compare their equations, noting how they are alike and different.

Call the class together to share some of the ways they grouped the paper clips or tiles. Find one pair who made arrays and ask them to make their sketch on the board. Ask:

How many rows did you use? What did you do with the extra items that didn't make up a full row?

Who has another way to write an expression for this arrangement?

Did anyone put the paper clips or tiles in groups of the same size? What size groups did you use? Why?

Which was easier for you to count? Why?

In the sharing, highlight equivalent expressions and equivalent equations. If equations or expressions are incorrect, show how *you* see what the student's equation represents pictorially.

Move students to *Part 2 (Student Book*, p. 157), where students draw pictures to match mathematical expressions. Pairs of students select two expressions to draw, and those pairs who finish quickly might draw a picture for all four expressions.

Students share and compare their pictures, checking to see that the pictures correspond to the expressions.

See *Lesson 10 in Action*, p. 128, for ways students connected expressions and visualizations.

By the end of *Activity 2,* students will have used objects to create arrangements which they then wrote as mathematical expressions and drew as pictures to represent expressions.

Activity 3: Garden Pathway

Refer students to *Activity 3: Garden Pathway* (*Student Book*, p. 158).

When everyone is clear about the directions, allow time for individuals to create at least two ways to show the number of tiles mathematically. Then bring the class together to share expressions.

There are several ways that students might write the expression for 68 tiles. Some might see the double rows of two 10s on the sides and the corners with four tiles each. Possible answers include:

1. a row of 14 on top, a row of 14 on bottom and 4 rows of 10 on the sides

   $14 + 14 + 10 + 10 + 10 + 10 = 2(14) + 4(10)$

2. 2 rows of 12 on right and 2 rows of 12 on left and a row of 10 on top and a row of 10 on bottom

   $12 + 12 + 12 + 12 + 10 + 10 = 4(12) + (2 \times 10)$.

Some students may figure out the number of tiles to cover the whole area, and subtract those that cover the garden:

$14(12) - 10(10)$ or

$14(12) - 10^2$

If no one brings up the subtraction, say:

I saw this as 14(12) – 10(10). What did I see?

During the review, insist that students demonstrate to one another how they came to their expressions.

Conclude by having them evaluate each expression, using the rules of order to verify that each expression equals 68.

Activity 4: Understanding the "Why" of Our Multiplication Methods

In this activity, students will have the opportunity to examine the mathematical principles behind their procedures, or algorithms, they regularly use for this operation. This is not a lesson on *how* to multiply with paper and pencil. It is assumed that students have learned to do this. If there are some who do not know how to, this discussion will uncover that, and you may want to provide extra practice.

Part 1: Students share their algorithms for multiplication

Launch from the algorithms that student use with a public display of "Our Multiplication Procedures (Algorithms)," similar to those suggested for addition and subtraction in *Lesson 5*.

Start by saying:

I know you have been doing basic multiplication for a long time, sometimes in your head, sometimes using a paper and pencil, and sometimes with a calculator or computer. It depends on what the situation is.

Right now, we're going to take a look at the paper and pencil methods you learned in school, and may still use, and discuss not just how they work, but why they work, connecting them to the arrays we have been working with. This knowledge is important groundwork for higher math.

Present the following problems to solve:

What is the cost of 13 T-shirts @ $11 per shirt?

Your three-year cell phone contract is almost up. For the last 32 months, you have paid $45/month. What have you paid on the contract thus far?

First ask the class for estimations, and record those on the board.

How would you figure out the amount using paper and pencil?

Ask students to check with another person to see if they did it exactly the same way. Then have volunteers show various methods on the board, explaining how they did it.

Heads Up!

Students from other countries may not have learned the U.S. standard algorithm, but might still use a standard algorithm that is based on sound mathematical principles.

Part 2: Probing *why* school methods work mathematically

After methods are posted, direct students' attention to seeing multiplication with arrays, by completing and discussing:

- *Math Inspection: Rectangles, Arrays, Area, and the Distributive Property* (*Student Book,* p. 160), using single-digit rectangular arrays to visualize how the property works; and,
- *Math Inspection: Connecting Arrays to Multiplication (Student Book,* p. 164), using two-digit rectangular arrays to visualize how the U.S. standard algorithm for multiplication works. This is not designed to teach students how to multiply, but rather to show them why the algorithm works.

After completing the math inspections, return to the Public Display of "Our Multiplication Algorithms." Ask students to work in pairs to look for connections between the way they did 11 × 13 on paper and the array on p. 164, and between the way they did 45 × 32 and the array on p. 166.

Math Inspection: Rectangles, Arrays, Area, and the Distributive Property

This inspection illustrates the distributive property. For example, the first situation, (6 × 7), has been regrouped to show 6(5 + 2), or 6(5) + 6(2). Seven could have been regrouped as 6 + 1, 3 + 4, or some other combination totalling 7. Being able to break amounts apart makes it easier for problem solvers to use the math facts they know, which lessens the cognitive burden of mental math. If, for example, the student doesn't know the product of 6 × 7 is but knows his 5 facts, breaking the 7 into 5 + 2 allows him to do the multiplication mentally.

In this inspection, ask students to work in pairs. Provide them with graph paper so that they can easily sketch the array. Encourage them to come up with different strategies for breaking apart numbers so that they can see that, no matter how the amount is separated, the total (product) is always the same. Be explicit in illustrating the distributive property, that is, no matter how 6 or 7 is divided, there is still distribution of amounts. For example, in 6 × (3 + 4) = 6(3) + 6(4), the operation of multiplication has to touch both parts of 7: 3 and 4.

Math Inspection: Connecting Arrays to Multiplication

The previous inspection focused on using arrays to illustrate the distributive property. This math inspection continues to build on this idea, and in doing so it clarifies why the standard multiplication algorithm works with multi-digit numbers. The point of this activity is to have students regroup using place value. For example, the number 25 would be regrouped into 20 + 5 to focus on two tens and five ones.

Once the numbers are regrouped, students compare the steps and resulting products with each of the products in the array. This should help students see the way the standard algorithm works. In the example 11 × 13, the amounts being multiplied are (10 + 3) × (10 + 1) … . Therefore, the 3 is multiplied by 10 and by 1. The array clearly shows these sub-products [(10 × 10), (10 × 1), (3 × 10) and (3 × 1) so students can understand why they sometimes need "to move a space to the left" when they multiply and then eventually add everything together.

Summary Discussion

Refer students to *Reflections* (*Student Book*, p. 263).

After students have written their responses, take statements from volunteers.

Give students time to write responses to these questions:

1. By sharing our different methods, what did we learn from one another in this lesson?
2. Many people say, "I just know how to do the procedure—I don't know why or what it really means." What do you understand clearly about why your multiplication procedure works the way it does?
3. What is still confusing?

Have students record their definitions in *Vocabulary* (*Student Book*, p. 255).

Practice

Number of the Day, p. 168
For practice finding factor pairs for the number 120.

Cartons of Eggs, p. 169
For practice writing equations to match arrays.

Expressions, Arrays, and Stories, p. 170
For practice connecting arrays with various expressions and relating stories to each array.

How Do You See It? p. 171
For practice writing equations and finding totals without counting.

Stone Paths, p. 173
For practice with rectangular arrays.

Sketch the Two Expressions, p. 176
For practice sketching two expressions to show that they are equal, illustrating why the distributive property works.

Mental Math Practice

Square Numbers, p. 177
For practice squaring numbers and using exponential notation.

Extension

Seeing Squarely, p. 178
For practice visualizing arrangements for expressions that include exponents.

Missing Rolls of Film, p. 179
For a challenge counting objects in a three-dimensional arrangement.

Test Practice

Test Practice, p. 180

Looking Closely

Observe whether students are able to

Write math expressions to reflect regular arrangements of objects in groups and arrays

Can students find the total number of objects in such an arrangement without counting (e.g., by adding or multiplying)?

Can students explain their approaches with reference to arithmetic and record their approaches using equations? Encourage them to explain their thinking using words such as "multiply" or "add." Say, for example, "You knew that there were three rows and each row had six squares. What did you do to find that there were 18 squares in all?"

Help students become familiar with the appropriate arithmetic terms: "So, you knew that three 6's are 18—you multiplied 3 by 6 and got 18."

Refer students to equations you (or others) recorded for the class and, as needed, model correct recording yourself. Some students may be familiar with different ways of recording operations and equations. For this lesson, encourage them to record in ways that are comfortable—as long as they are correct—but also point out alternative notation, e.g., $3 \times 4 = 3(4) = 3 \cdot 4$.

Ask those students who struggle to explain how they are counting the objects or to count aloud for you so you can repeat or record for them what they said. With a pencil, mark off the sets of objects to reflect how students are seeing them.

Do students notice commutative relationships among operations—in other words, that three rows of four is the same as four rows of three?

Represent equations using arrays and/or equal groups arranged to correspond with numbers and operations

Do students recognize that arranging objects into equal groups makes it possible to find the total without counting each object?

Ask students to arrange the items into groups of a particular size (say, four or five items to a group) and then to find the total number of items without counting each item one by one. If needed, work with students to add or skip-count to find the total: "Okay, we have four groups of five items. So, this first group has five items; how many items are in these two groups together? How many are in three groups?"

Are students able to find a total number in an array by multiplication? As above, ask students to model the number with objects in groups of 10, and ask them to find the total by counting by 10's.

Can students make an abstract-looking multiplication problem concrete? Using arrays should make the key ideas of multiplication obvious—an accumulated total of equal groups. The algorithm for finding the product of two- and three-digit numbers should also make more sense once students have seen how an array can be broken apart into smaller multiplication problems whose products are then totaled. Three properties play a role: commutative in that any order or set up of the array is as acceptable as another (3×4 vs. 4×3); associative in that the numbers can be broken down (into 10's and 1's for example); and the distributive (that the same number, the multiplicand, acts on all the parts of any number that has been broken up).

Identify equivalent expressions

Are students able to verify that equations are equivalent or nonequivalent by demonstrating with visual models? Take time to connect the visual models with the mathematical symbols.

Are they able to use rules of order to evaluate an expression? Return to *Lesson 9*, where order of operations was a main focus.

Rationale

Arrays provide a clear picture of multiplication and open the way for visualizing multi-operational equations. The lesson provides plenty of opportunity to practice multiplication facts, but the emphasis is on the conceptual understanding of multiplication.

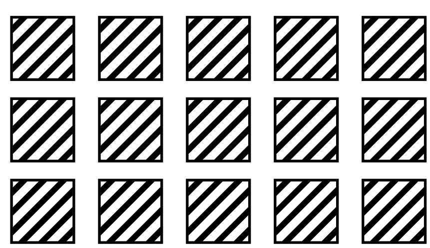
This lesson also helps to build flexibility with breaking apart and putting together numbers, an essential skill throughout mathematics.

Math Background

Arrays are handy for organizing multiplication because rows and columns make it easy to see numbers of equal groups. Yet some students will not trust that multiplying rows and columns will yield the same result as counting by 1's. Are students persuaded that they can break apart an array, combining sub-totals of smaller multiplication problems to come up with a product? Start with a manageable area of 12 or 16 if needed. Use colored pencils or different pens to keep track of which parts of the array have been counted and calculated in a running total.

Arrays lend themselves to a discussion of commutativity; one can rotate the array to see the rows and columns change, but the total remains the same. Arrays are generally more useful than groups for quick totaling because arrays show at a glance that they contain equal numbers of items; groups do not.

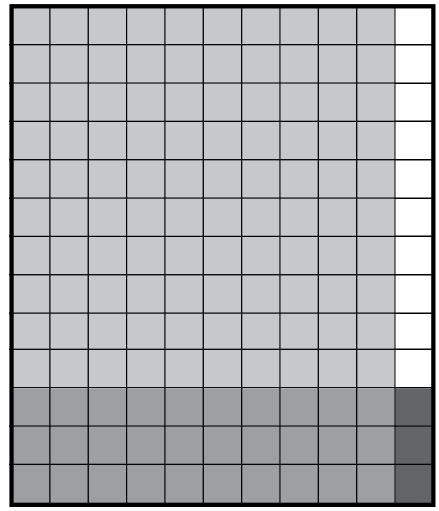
Some students may be comfortable with commutativity in addition and multiplication. They may take it for granted that 40 + 7 is the same as 7 + 40, or that 3 × 5 is the same as 5 × 3. Others may be less certain and may benefit from a visual approach. For instance, suppose students are explaining how they know how many squares are in the following array:



One student may see this as three rows of five, or 3 × 5. Others may see it as five columns of three and write "5 × 3." With repeated experiences such as these, over time students will gain understanding of commutativity in multiplication and addition.

The algorithm for finding the product of two- and three-digit numbers should also make more sense once students have seen how an array can be broken apart into smaller multiplication problems whose products are then totaled. Three properties play a role: commutative in that any order or set up of the array is as acceptable as another (3 × 4 vs. 4 × 3); associative in that the numbers can be broken down (into 10's and 1's for example); and distributive (that the same number, the multiplicand, acts on all the parts of any number that has been broken up).

Using arrays to understand the partial sums in the standard multiplication algorithm (*Math Inspection: Connecting Arrays to Multiplication*) may be new for some teachers. The array model for two digit times two digit multiplication shows visually the four multiplication steps involved as the four sections of the larger array. The visual model not only shows why the procedure makes sense, but it also makes place value prominent. If we look at the example from the *Student Book*, p. 164, many students talk themselves through the algorithm without attention to place value: "One times three is three, one times three is three, put down a zero …" The array model reminds them that not all the 1's are really 1: 1 × 3 is 3, but the next sub-product is actually 10 × 3 = 30. The array makes this clear.



Facilitation

The more time devoted to notation, the longer this lesson will take.

Some students take the direction to sketch literally. Recommend the use of sticks, X's, or dots as ways to sketch a representation of an arrangement.

To connect the lesson more firmly with daily life, take some time to examine objects and photographs of real arrays and discuss how students see the total in each.

Do an internet search for images of architecture, items packaged in sets, items arranged for sale or storage, and art that involved repeating geometric patterns.

Making the Lesson Easier

Assign fewer sets in *Activity 1: Pictures and Numbers* (*Student Book*, p. 152) and limit selection to the simpler arrangements. In *Activity 2*, distribute fewer paper clips to student pairs.

Making the Lesson Harder

Discuss square and prime numbers with students. Assign *Extension: Seeing Squarely* (*Student Book*, p. 178) as a challenge.

Teachers commented on the value of connecting expressions with visual representations.

When drawing out the situations for a number sentence, it was interesting to find that students see multiplication as addition. Having to illustrate a situation made them really think about what it meant. Once I suggested that they try to think a little differently, they began to change their illustrations. This has such potential really to bring so many big ideas of multiplication to the surface.

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This lesson worked fairly well as an introduction to seeing and discussing multiplication. Students were immediately engaged in counting the numbers of things in each photo. The discussion went well, but there are such huge gaps in how people approach this problem (from counting one by one to counting by groups, and *not* often by multiplication). Making the pictures for the equations was equally as engaging, though the frustration level was higher. Again, multiplication was definitely seen as addition.

One student began to see that three groups of three was also one group of nine. That was big to me, but the concept was a slippery one for a while and I'm not so sure that he solidly understands it.

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EMPower Plus

extending mathematical power

Everyday Number Sense

Mental Math and Visual Models

STUDENT BOOK

LESSON 10

Picture This

How many are there?
How do you know?

Counting is important in our everyday lives. Knowing how to total the items in a group efficiently is valuable because it saves time.

In this lesson, you will look at familiar objects arranged in groups, and you will be asked to find the total without counting each object. You will also write mathematical expressions that show that value.

For instance, if there are two six-packs of soda, you might write this:

6 + 6 6(2) *or* 2(6)

These are **equivalent expressions**; their value, 12, is the same.

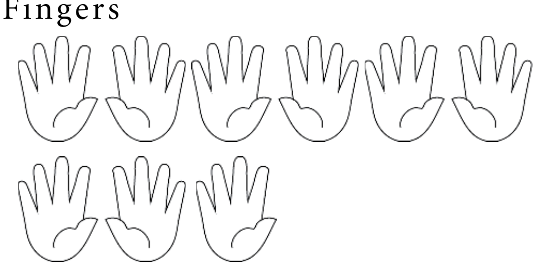
Activity 1: Pictures and Numbers

Part 1

Choose one picture from this page and one from the next page. For each, find the total number of objects in the picture, but don't count one by one. Write down two or more ways to find the total.

1. Soda Cans

2. Fingers



3. Heads

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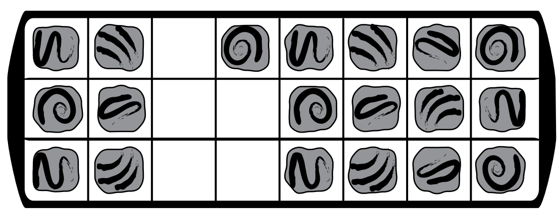
4. Stamps

5. Chocolates



Part 2

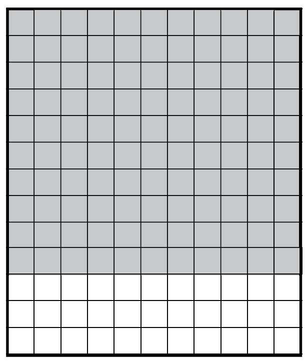
Each of the following expressions describes one of the arrays drawn below. Find the expression that best matches each **array** and write its letter on the line provided. Pay attention to the way the squares are shaded.

a. $13 \times 10 + 13 \times 1$

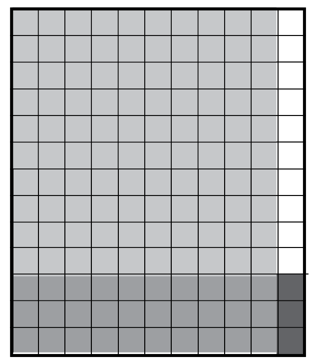
b. $10(11) + 3(11)$

c. $10(10) + 10(1) + 3(10) + 3(1)$

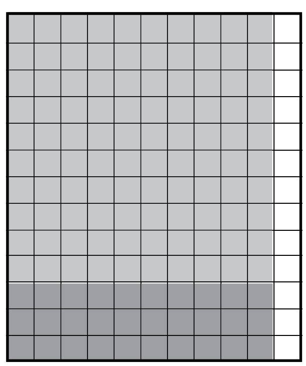
d. $10 \times 10 + 13 \times 1 + 3 \times 10$



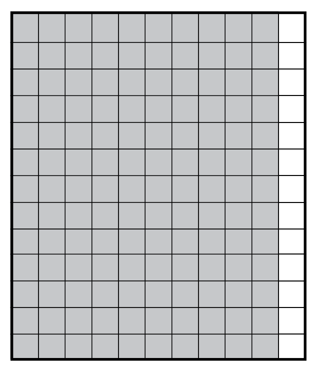
1. Expression: __________



3. Expression: __________



2. Expression: __________



4. Expression: __________

5. **a.** $13 \times 11 =$ __________

b. How did you find the answer?

If you used yet another array to help you find the answer, sketch it below.

6. Explain why the four expressions on page 154 are equivalent expressions.

Activity 2: Counting Smart

Part 1

Take a handful of paper clips, pennies, or tiles. Arrange them as arrays so you can see how many there are without counting each one.

1. Sketch your arrangement using columns and rows.

2. Write an equation that shows how you can find the total amount without counting each item.

3. Sketch another arrangement. If you did not try arranging by 10's, try that now.

4. Write an expression that shows how you found the total amount without counting each item.

Part 2

Draw a picture that communicates each mathematical expression.

1. $4 \times 9 + 2$

2. $20 - 3 \times 5$

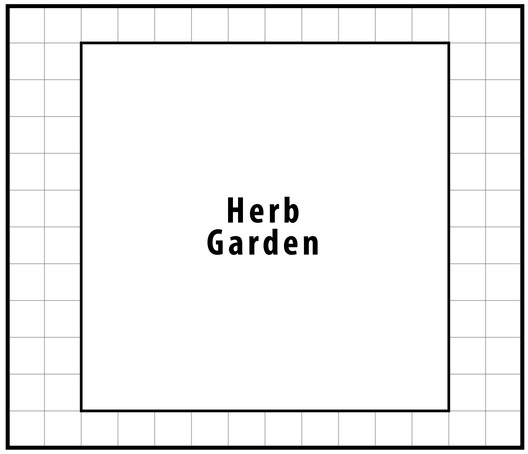
3. $3 \times 4 \times 5$

4. $3(4 + 6)$

5. Now pick one of the mathematical expressions above and write a word problem for it.

Activity 3: Garden Pathway

Valerie and Rebecca own a landscaping business. A customer wants them to install a garden and a pathway made of square tiles surrounding it. This is the picture the customer provided.



Each woman saw the math differently. Of course, they didn't count each tile! Show two different ways that Valerie and Rebecca could have figured out the number of tiles.

1. First way:

2. Second way:

Activity 4: Understanding the "Why" of Our Multiplication Methods

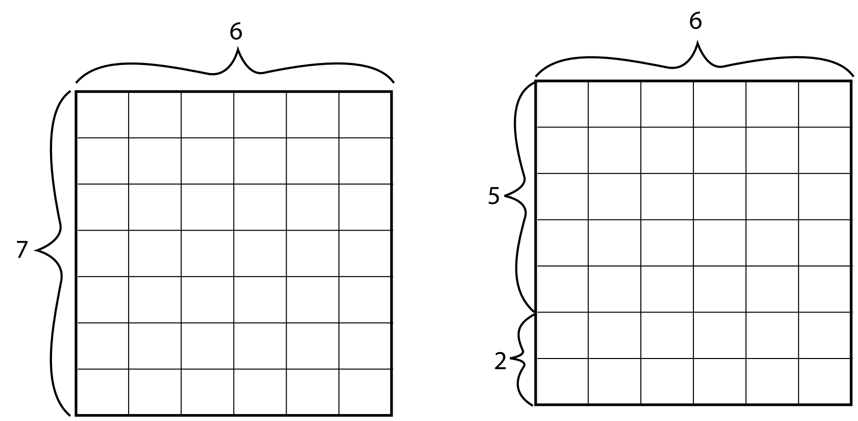
1. What is the cost of 13 T-shirts at $11 per shirt?

2. Your cell phone contract is almost up. For the last 32 months, you have paid $45 per month. How much have you paid on the contract thus far?

Math Inspection: Rectangles, Arrays, Area, and the Distributive Property

One way to show multiplication is with a **rectangular array**.

1. Look at this example:

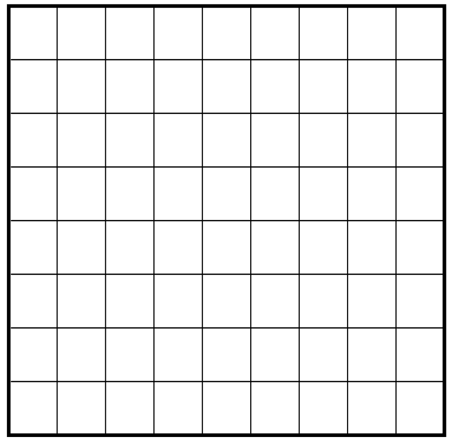


a. Explain in words what you see happening.

b. Explain in mathematical symbols what is happening.

2. Below is an 8 × 9 array.

a. Break it up into a new multiplication problem.

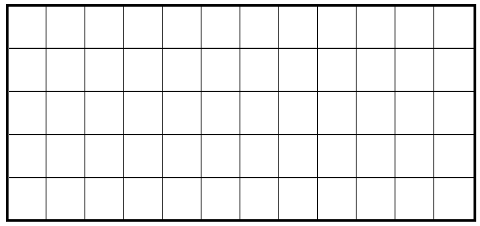


b. Explain in words what you see happening.

c. Explain using mathematical symbols what is happening.

3. Below is a 5 × 12 array.

a. Break it up into a new multiplication problem.



b. Explain in words what you see happening.

c. Explain using mathematical symbols what is happening.

4. Fill in the missing numbers below.

a. $6 \times 7 = 6 \times 4 + 6 \times$ ______

b. $9 \times 8 =$ _____ $\times 8 + 4 \times 8$

c. ____ $\times 7 = 2 \times 7 + 5 \times 7$

d. $6 \times 7 = 5 \times 7 +$ _____ $\times 7$

e. $7 \times$ _____ $= 5 \times 6 + 2 \times 6$

f. $9 \times 9 = 5 \times 9 +$ _____ $\times 9$

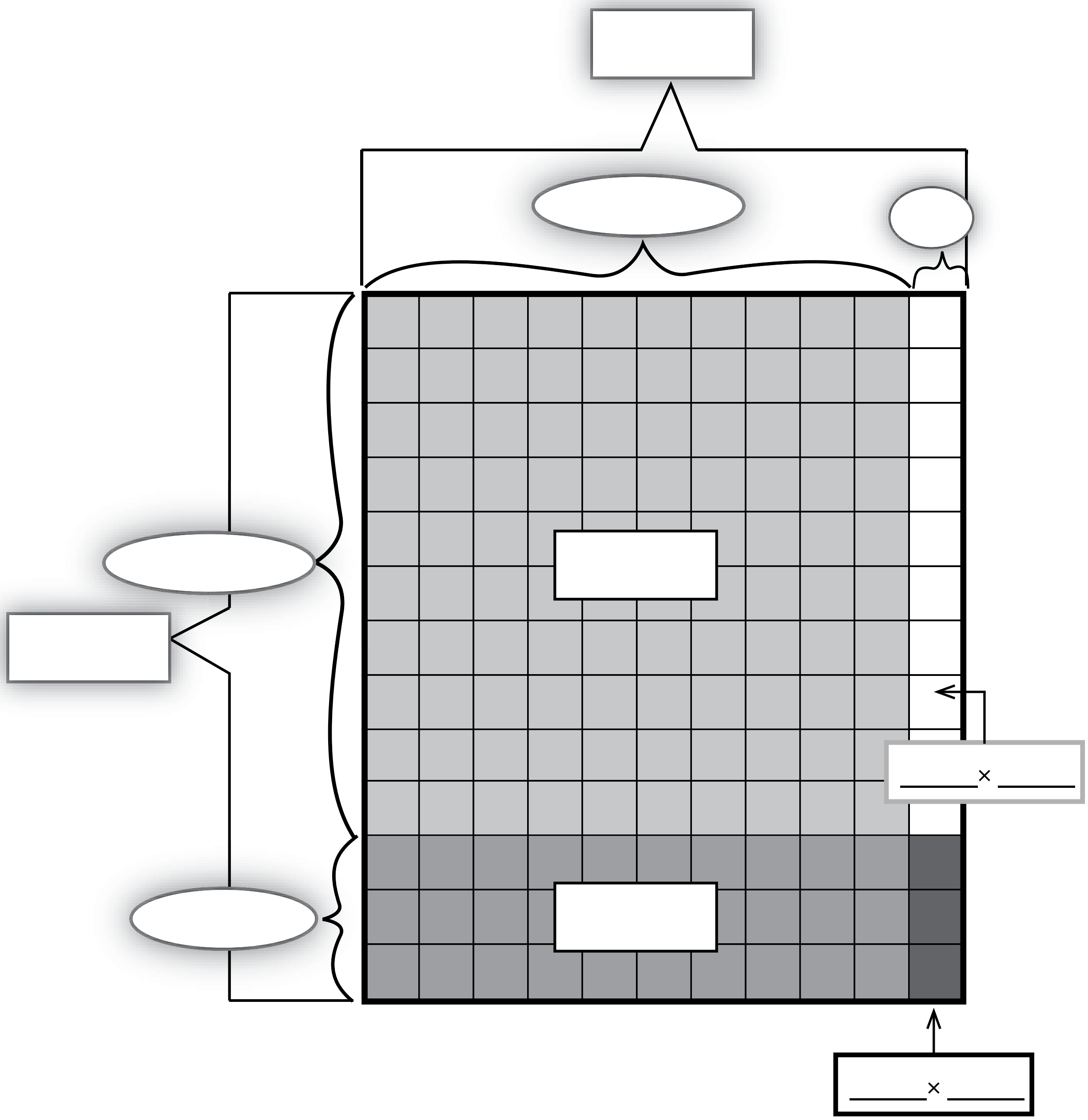
g. $8 \times 7 =$ _____ $\times 7 + 5 \times 7$

Math Inspection: Connecting Arrays to Multiplication

Take a close look at how an array connects with the way many of us learned to do multiplication. Earlier you examined the problem 11 × 13. One way to draw that situation is shown below, where 11 × 13 is understood as (10 + 1) (10 + 3).

```
   11
 × 13
 ----
    3 ... 3 × 1
   30 ... 3 × 10
   10 ... 10 × 1
  100 ... 10 × 10
 ----
  143
```

1. Label each of the parts below. Fill in all the missing numbers to show how parts are being multiplied. Then add the parts to get a final product.

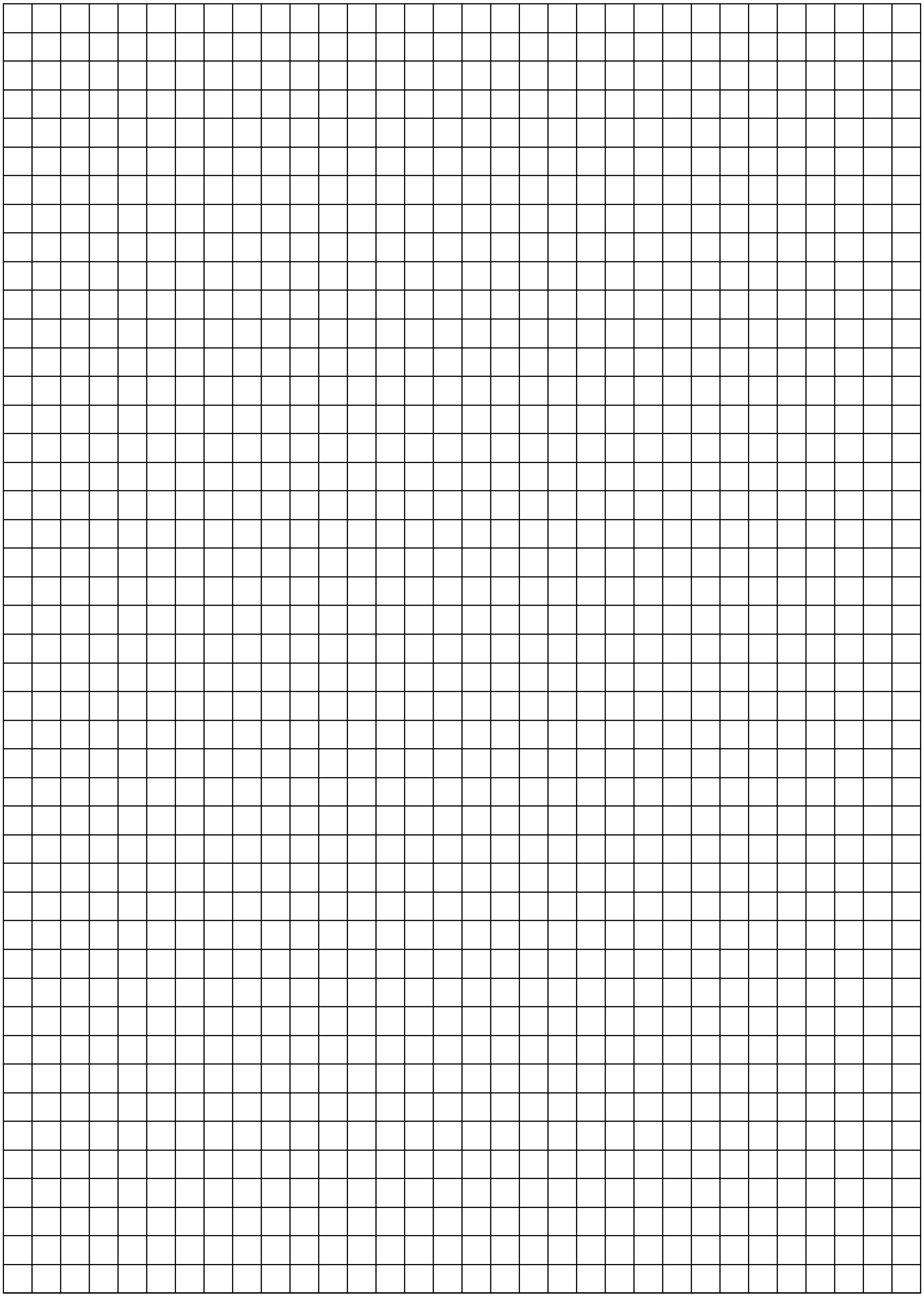


a. Explain in words what you see happening.

b. Explain in symbols what is happening.

c. Describe all the connections between the array and the solution. How many sub-products can you see? Where is addition?

2. Below is a 45 × 32 array.



a. Break up the numbers to show the multiplication problem: (40 + 5)(30 + 2). Write all of the sub-products you see, as well as the final product.

b. Which connections are easy to see? Which are confusing? Why?

c. Write a rule that says how you would use this idea with *any* pair of numbers.

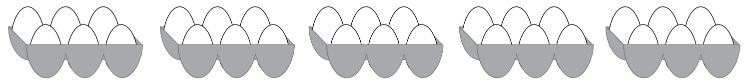
Practice: Number of the Day

Number of the day : 120

Focus on writing expressions with two **factors** that equal 120. Write as many ways as you can. Then use the same numbers and switch to writing equations using division.

> *Reminder*: Factors are the one of two or more whole numbers that can be multiplied together to give a specified number. For example, 1, 2, 3, 4, 6, and 12 are the factors of 12 because 1 × 12, 2 × 6, and 3 × 4 = 12.

Practice: Cartons of Eggs



1. Without counting each egg, how many do you see?

2. How did you think of your answer?

3. List with words and numbers each step you took mentally or on paper to find the total.

4. Write another expression to show how you could count the eggs.

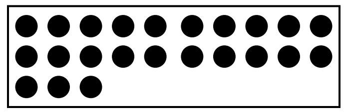
Practice: Expressions, Arrays, and Stories

Part 1

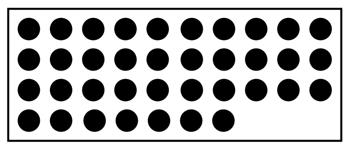
Circle the expressions that do not match the picture.

Reminder: Parentheses indicate multiplication or tell you to do the operation inside them first.

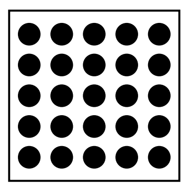
1. $10 + 10 + 3$ $3 \times 3 + 10$ $10 \times 2 + 3$ $3 + 2(10)$



2. $3 \times 10 + 7$ $10 + 10 + 10 + 7$ $3(10 + 3)$ $4 \times 7 - 3$



3. $5 + 5 + 5 + 5 + 5$ $5(5)$ $5 \times 5 + 5 + 5 + 5$ 5^2



Part 2

Match each story to one of the pictures above.

4. Zippy and four friends combine their money. Each person gives the same amount, $5.

Array ______

5. Zippy and two friends want to buy a gift for their teacher. The two friends can spend $10 each. Zippy has $3.

Array ______

6. Zippy and three friends order take-out. Everybody chips in $10 but Zippy. He is $3 short.

Array _____

Practice: How Do You See It?

For each picture below, give the total number of shapes or coins, *without counting one by one.*

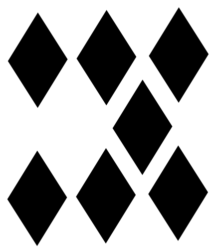
Circle the groups you see in each picture.

Write two different expressions to show how you could find the total.

In one expression use at least two operations (for example, multiplication and addition or addition and subtraction)

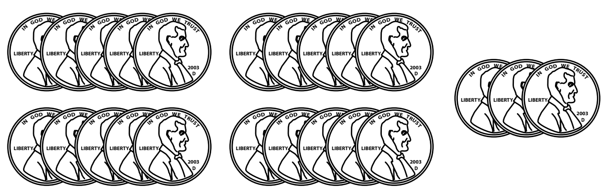
1. Circle the groups you see in each picture as you figure out the total. Write two expressions that describe what you see.

a.



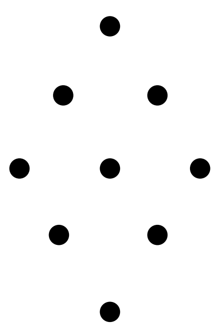
Expression 1: ________________ Expression 2: ________________

b.



Expression 1: ________________ Expression 2: ________________

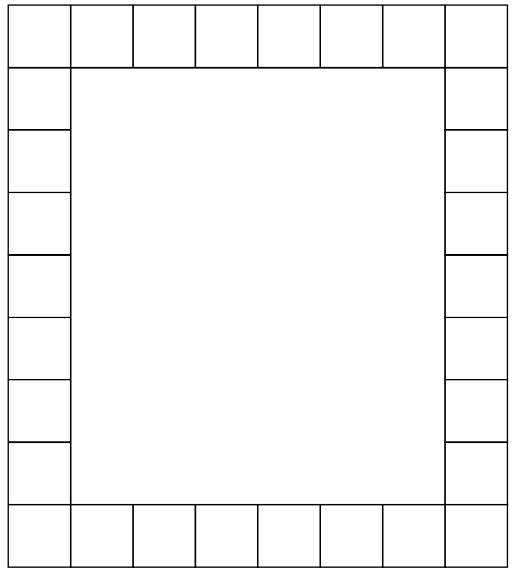
c.



Expression 1: ________________ Expression 2: ________________

2. Show with words or an expression how you found the total number of tiles.

a.



Words/Expression:

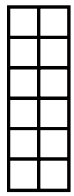
b. Show with words or an expression how you could find the total another way.

Practice: Stone Paths

The following are rectangular arrays for paths made using 12 flat stones.

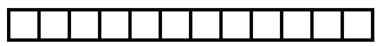
1. For each one, write an equation that describes the array.

a.



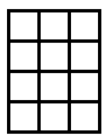
Equation:________________________

b.



Equation:________________________

c.

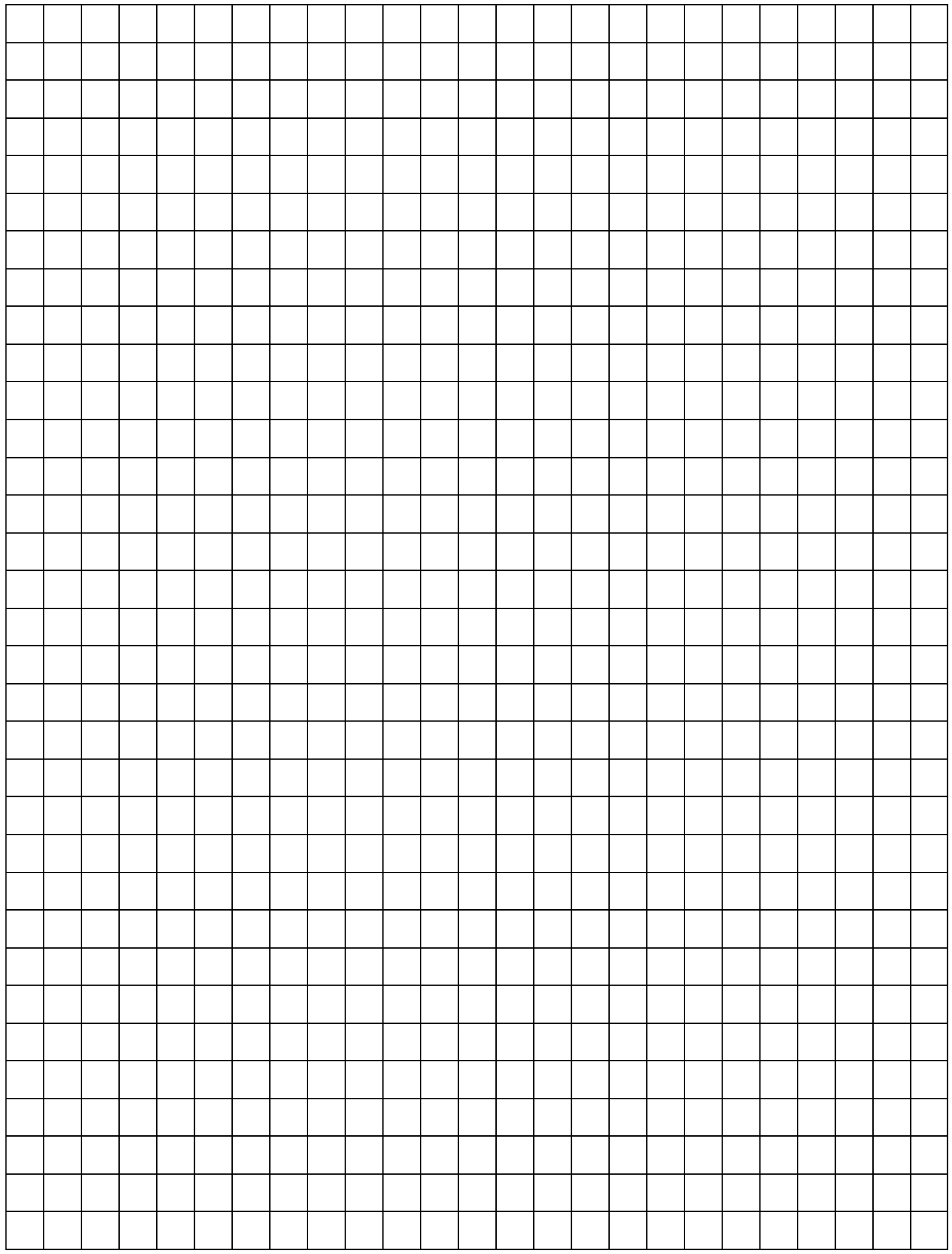


Equation:________________________

2. a. Find all the possible stone path arrays for 23 rectangular stones. Do the same for 24 and 25 stones. Use the grid paper on the next two pages to draw the arrays.

b. Write equations that describe each of the arrays you find.

c. Why are there more paths for 24 stones?



Practice: Sketch the Two Expressions

For each example below, use a representation to show why the two expressions are equal.

1. $2(3 + 4) = 2\ (7)$

2. $4(2) – 6 = 8 – 6$

3. $8 – 2(3) = 8 – 6$

4. $3(5 – 2) = 3(5) – 3(2)$

5. $3(5 – 2) = 3(3)$

Mental Math Practice: Square Numbers

You **square** a number by multiplying it by itself. You can use an **exponent** (a small, raised number) to show the multiplication. For example, 5 × 5 can be written as 5^2. You read that as "five squared."

1. Square all the whole numbers from 0–12.

n	0	1	2	3	4	5	6
n^2							

n	7	8	9	10	11	12
n^2						

2. Fill the missing numbers in the boxes to make each equation true.

a. $\square \times \square = 36$

b. $\square \times 9 = 81$

c. $9 = \square^2$

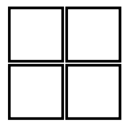
d. $8^2 = \square$

e. $0^2 + 1^2 + 2^2 + 3^2 = \square$

Extension: Seeing Squarely

Square numbers can look like square-shaped arrangements. Draw an arrangement of objects for these expressions:

Example: 2^2



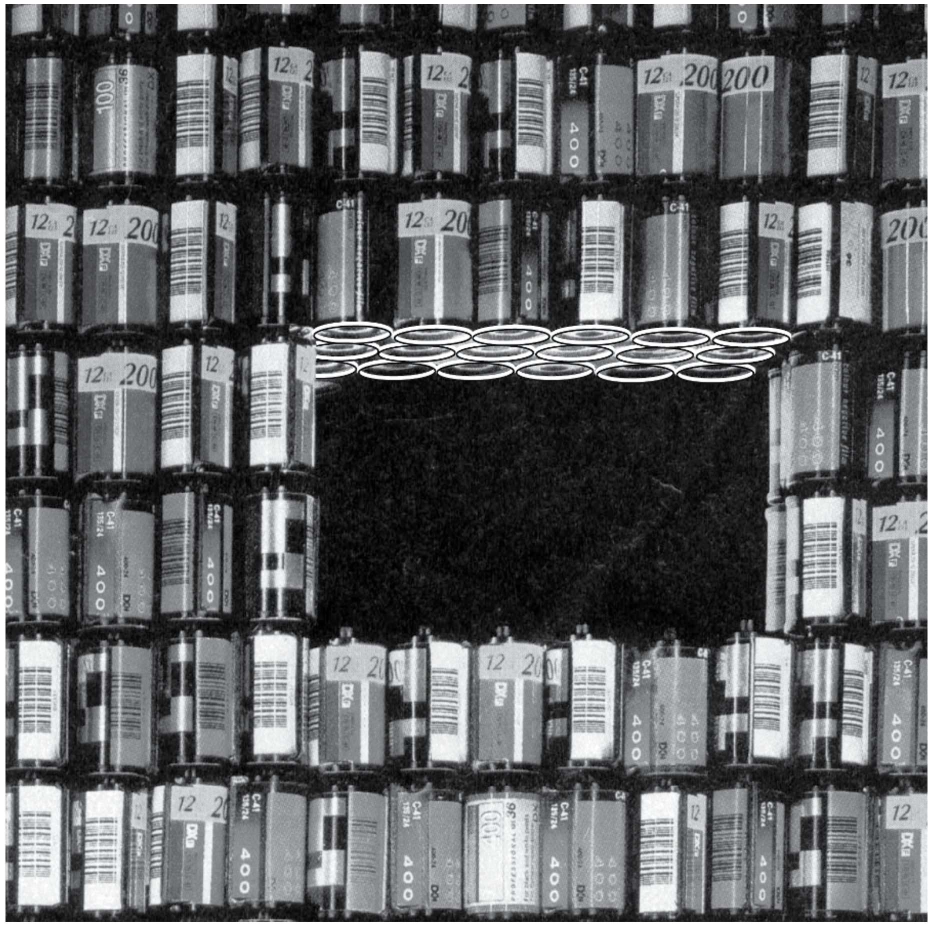
1. 5^2

2. $5^2 + 3^2$

3. $(5 + 3)^2$

4. $5^2 - 3^2$

Extension: Missing Rolls of Film

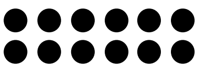


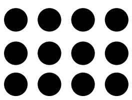
1. How many rolls of film are missing in the black area?

2. How do you know?

Test Practice

1. Which of the following arrays matches the equation $3 \times 4 = 12$?

   A. 

   B. 

   C. 

   (a) A

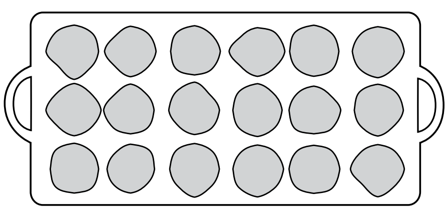
   (b) B

   (c) C

   (d) A and B

   (e) B and C

2. Which of the following expressions might be used to count the cookies on the tray?

   A. 6+ 6+ 6 **B.** 6×3 **C.** 6×6

   (a) A only

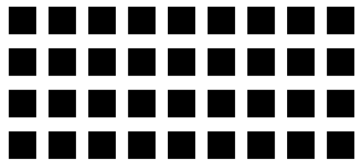
   (b) B only

   (c) C only

   (d) A and B

   (e) A and C

3. Frankie needs 45 tiles to cover the bathroom floor in her apartment. The picture below shows how many she has already installed. Which of these expressions shows the number of tiles Frankie still needs to install to finish the job?

   (a) 4×9

   (b) $45 + 4(9)$

   (c) $4(9) \times 45$

   (d) $45 - (4 \times 9)$

   (e) $(45 - 4) \times 9$

4. Select the expression that is *not* equivalent to the rest of the expressions.

   (a) $36 + 4(5)$

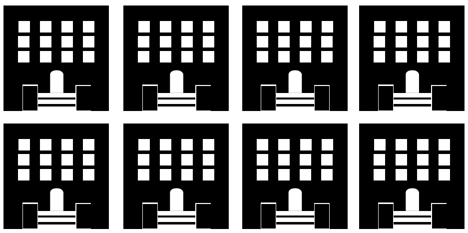
   (b) $4(9) + 20$

   (c) $3(12) + 20$

   (d) $4(9) + 4(6)$

   (e) $2(18) + 2(10)$

5. The mayor wants all of the front windows on the city's eight libraries to be cleaned. Charlie's Windows charges $6 per window for cleaning. Which of the following expressions shows how much Charlie's Windows will charge the city?



(a) $6(3 \times 4)$

(b) $6 + (3 \times 4)8$

(c) $8(12 + 6)$

(d) $6(4 \times 12)$

(e) $6(3 \times 4 \times 8)$

6. Lois enters a party room and sees people sitting at round tables. Eight people are seated at each of 12 tables and five people at each of two other tables. How many people does Lois see?