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Many Points Make a Point

Data and Graphs

TEACHER BOOK

EMPower Product Sampler Correlations for Many Points Make a Point: Data and Graphs

EMPower Mathematics Topic Correlation Guide			
Many Points Make a Point: Data and Graphs			
Book Description: Students collect, organize, and represent data using frequency, bar, and circle graphs. They use line graphs to describe change over time. They use benchmark fractions and percents and the three measures of central tendency—mode, median, and mean—to describe sets of data.			
Lesson Number:	**Lesson Name:**	**Mathematical Concepts/ Topics Covered:**	**Pages (Teacher Book):**
Opening the Unit	Many Points Make a Point	• Assess familiarity with **graph formats, features, and purposes** • **Graph terms**	1–14
Lesson 1:	Countries in our Closets	• **Categorize** and **compare data** with frequency graphs • Identify **graph 'story'** • Change **data display** to see change in graph 'story'	15–26
Lesson 2:	Most of Us Eat	• **Organize data** for specific purposes • **Describe data** numerically with benchmark fractions and percents	27–38
Lesson 3:	Displaying Data	• **Bar and circle graph construction** • **Axes intervals** • **Bar and circle graph formats** compared and contrasted	39–48
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Interim Assessment	The Data Say	• **Bar and circle graph knowledge assessed**	61–70
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Lesson 7:	A Mean Idea	• **'Average' (mean)** defined and determined given all values or missing values	91–100

Lesson 8:	Mystery Cities	• **Multiple data lines** • **Scale variation impact** • **Graph and text alignment**	101–110
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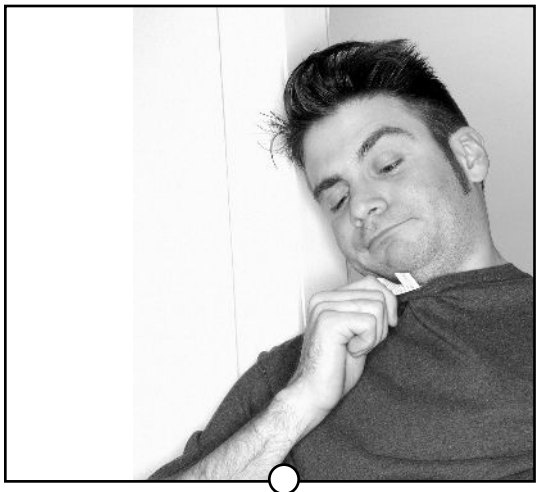
Lesson 10:	Stock Prices	• **Tables** connected to and generated from graphs • **Scale generalizations**	121–132
Closing the Unit	Stock Picks	• **Application of graph knowledge** for evaluations, recommendations, problem solving and presentations	133–140

FACILITATING LESSON 1

Countries in Our Closets



Where were your clothes made?

Synopsis

This is the first of a set of lessons on categorical data in which students collect, examine, sort, and organize their own data, and then compare their data to another data set.

1. Students bring to the class data on the different geographic locations where their clothes were made.
2. The class combines its data into one frequency graph, and students make several statements about it.
3. Working in pairs, students compare their class graph to data from another class.
4. Pairs of students each create a new frequency graph, organizing the class data by continent. Students notice which continent is most prevalent.
5. Discussion centers on comparing country and continent graphs, closing with the question How does the story change when the graph's categories change?

Objectives

- Use a frequency graph to organize data
- Identify the story that the data tell
- Compare data from various samples
- Change the categories and articulate the change in the story

Materials/Prep

- Grid paper
- Paper strips for posting statements
- Sticky notes
- Rulers

As an aid for determining in which continent the countries are located, have access to a world atlas, if available. For a listing of countries in Asia, see *Resource: Countries in Asia, Blackline Master 4*.

Make sure you have assigned *Countries in Our Closets* (*Student Book*, p. 7). Collect and bring in about 20 items of clothing in case your students have not collected their own data. (See *Facilitation*, p. 25, for how to handle this.)

Prepare to display the graph "Countries in Our Closets, Springfield, MA" (*Student Book*, p. 10) during the opening of Activity 2.

Opening Discussion

Ask students:

Are clothes made in this city? In this country? Why or why not?

Why would anyone care where his or her clothes were made?

Clothing is close to us, physically worn next to our skin, so we should want to know where it comes from, but sometimes we take it for granted. We need to know something about where our clothes come from because of human rights, political, economic, and environmental issues. We can consider, for example, the age, safety, and wages of people manufacturing clothes in countries with fewer worker protections; the economic implications of outsourcing; and the cost of fast fashion to the resources of our planet.

Heads Up!

If students do not bring in data, see *Facilitation*, p. 25.

You have looked in your closets, checked the tags on some of your clothes, and collected information. What did you learn?

Which country do most of the clothes you checked come from?

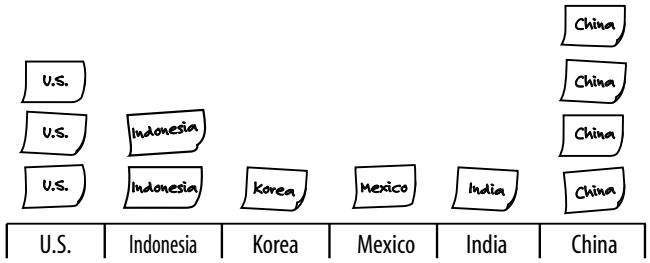
Post the country names in a place where you can easily refer to them later.

Introduce the terms **data set** and **mode**. Each student's list is a data set, and each data set could have a mode—the country name that occurs most frequently.

Vocabulary note: If students are searching for words to express their thoughts, offer terms and concise definitions. Students can keep track of these in their student books. For example, **data** are the information collected. The data that are collected make up the **sample**. The whole group from which information could be collected is the **population**. See *Lesson 1 Commentary*, p. 24, for more information.

Activity 1: Organizing the Data

Explain the motivation for organizing data: It is easy to identify the mode from a small sample, but when working with larger samples of data, it helps to have a strategy. In addition, a graph will illustrate the data, enabling everyone to see the number of articles of clothing from each country.



Distribute sticky notes to students. Ask them to write on a separate sticky note the name of the country each article of clothing came from. On the board, draw a horizontal line, long enough to fit many different country names. Ask students to place their sticky notes above the line, organizing the data by country. Volunteers should write the names for countries along the bottom of the line. This graph is called a **frequency graph** because it shows how often (frequently) each country occurs.

For distance learning, consider using a platform like Jamboard to collect and organize data for Activities 1-3

Heads Up!

Leave the whole-class frequency graph posted for use in the following activity. If you have a copier available, make and distribute to each student a copy of the class frequency graph done on graph paper.

If you do not have a copier, distribute graph paper for students to copy the whole-class data for themselves—be aware that this could be a time-consuming task.

When students finish, ask:

Which country did most of our clothes come from?

How does the mode for the whole-class data compare to the mode for your own data?

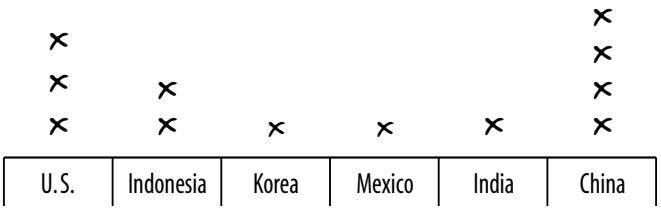
Ask students to create a few paired statements about the class data. Model this with a statement such as

One article of clothing was made in each of the following countries: China, Mexico, and Korea.

There are twice as many clothes from China as from Mexico.

Referring to the whole-class frequency graph, you might say:

All these sticky notes could fly off, and we would lose our data. One way to avoid that problem is to use x's in place of each sticky note like this (model it):

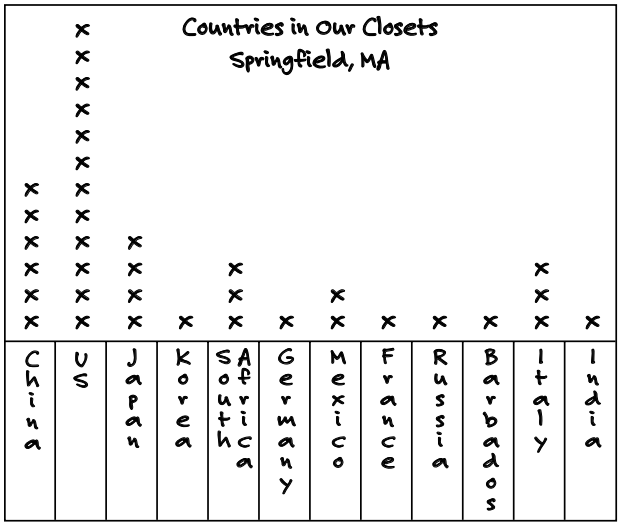


Note the components of a frequency graph:

- Line
- x's
- Labels

Activity 2: Statements about Data

Students now compare their class data with data from another class. Post the data and ask students to comment on similarities and differences between the country data from their class and from a pre-GED class in Springfield, Massachusetts.



Start by describing the Springfield data:

In the Springfield data, four times as many clothes come from the United States as from South Africa. Explain that statement.

As students explain, make sure "3 × 4 equals 12" is made explicit.

What other statements can be made about the Springfield data?

Prompt students to provide numerical examples (e.g., twice as many clothes came from China as from Italy; half as many clothes came from China as from the United States; nine more articles of clothing came from the United States than

from Italy). Refer students to *Activity 2: Statements about Data* (*Student Book*, p. 10). Ask them to work in pairs to answer these questions:

We have data from two sets of students—one in Springfield and our own. How are they similar? Different?

Do our two classes have the same mode? Why do you think that might be?

What if we asked students from another state to make a data set? How do you think their data would compare to these two data sets? Why?

What if we included people other than adults in our program? How do you think their data would compare to these two sets?

When researchers collect data, they ask themselves these kinds of questions.

Heads Up!

The important thing to note is that the statements made are valid *only* for the group from which the data are collected. We can hypothesize that other groups might have similar or different data, but we do not know for sure unless we collect data. This is meant to open a conversation about the concept of sample. Draw from the *Math Background*, p. 24, if your students are interested. *Lesson 2* addresses the issue of sample in more depth.

Activity 3: Changing the Categories

Next, the data will be grouped in another way—by continent. Students will work with the class data and might need additional reference materials. Refer students to *Activity 3: Changing the Categories* (*Student Book*, p. 11). Ask pairs of students to make a frequency graph showing the continents in their closets.

When you make a frequency graph, remember the three parts: the line, the x's, and the labels.

Ask the student pairs to post their graphs and to make one numerical statement describing the data in their graph. For example, "Less than half of the clothes came from Europe"; or "A quarter of our clothes came from South America."

What do you notice about this new way of sorting our clothes?

Be sure to highlight the following:

- It is often easier to remember and manage fewer categories.
- All the clothes from one continent could have been produced by many countries or by just one. The continent label hides this information.
- The "part of the world" many of our clothes came from becomes clear.
- The continent labels might change the relationships among categories. For example, the frequency graph by country could show that more than half the clothes were made in China; whereas the graph by continent could show that more than three-quarters of the clothes were made in Asia.

Open a discussion about the sample:

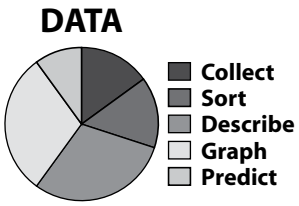
What do you imagine you would find if a different group collected and shared their data?

Would it be appropriate to make statements based on these data that would apply to all adults' clothes? Why?

Heads Up!

Generalizations from data are affected by how representative the sample is and by its size. You do not need to go into details about this now, but if students wonder about whether they *can* generalize, give an example such as "If all our data were collected in (*name a wealthy area*), the countries might not be the same as if we collected data from (*name a non-wealthy area*)"; or, "If we only looked at clothes bought at *(name a high-end clothing or a discount-fashion store*), we might be missing some countries."

Summary Discussion



Encourage students to share—orally or in writing—what they have learned about collecting, sorting, graphing, and grouping and regrouping data using a frequency graph.

Say:

You are in charge of making statements. The data are just pieces of information until you interpret them. The data stay static (that is, the data do not change)—the way they are represented does not. You can change the categories and/or combine them; this might yield a new story.

Circle back to the larger conversation about where the clothes were made. The data will likely show that most of our clothes were made far from home. Engage students in a conversation about why this is. You might jump-start the discussion by saying:

Although we have not talked about the cost of shipping or the cost of each item, companies seem to want to buy clothes made in other countries. Why might that be?

You might also pose the question, "What does 'Made in the USA' really mean?"

This is a good time to remind students about the three components of a frequency graph. Go over the steps they took to represent the data, i.e., collecting, sorting, organizing, labeling, and making statements and generalizations about the data. Direct students to *Reflections* (*Student Book*, p. 144), where they can record their answers.

Practice

Clothes by Continent in Springfield, MA, p. 12
For practice sorting data by continent.

Reporting Data 1 and 2, pp. 13–14
For practice seeing what effect collapsing data has on the story the graph tells.

Categorically Speaking, p. 15
For practice analyzing how things in our lives are organized in categories.

Extension

Taking Inventory, p. 16
For practice collecting new data, then sorting, categorizing, and reorganizing them.

Test Practice

Test Practice, p. 17

Looking Closely

Observe whether students are able to

Use a frequency graph to organize data

How do students manage the logistics of constructing a frequency graph? Make sure they use the grid lines as a guide so that that the x's are uniform. This way, when they compare countries, the comparison will be precise. Help students make the connection between one article of clothing and one x. Ask them to double-check their graphs to make sure they included every data point, not adding or omitting any. The category labels are also important. Assist students with spelling, if necessary.

Identify the story that the data tell

Do students make statements based on the data displayed on the graph? Some students might feel the job is done when the graph is complete. Not so. If students are hesitant, pose questions about what the data tell: "Is there one country that stands out in the graph? How so? Why might that be?" Accept either statements that quantify the relative value ("Asia has three times as many") or the total value ("India has nine more than …").

Compare data from various samples

Do students notice the big differences or similarities between two data sets? If they say, for example, "This data set includes Japan and the other one does not," that is a good start. Probe for more by asking, "Do they both have the same mode? How do you know?" You are only looking for a big-picture comparison.

Change the categories and articulate the change in the story

Do students recognize the differences between the frequency graphs and the accompanying stories for countries and continents?

While some of the graphs might look similar, the labels are different, as are the groupings. Ask students who have trouble articulating specific questions: "What are the categories (groups) in this graph? And in the other one? Are there continents that stand out? How are they related to the countries that stood out?"

WHAT TO LOOK FOR IN *LESSON 1*	WHO STANDS OUT? (LIST STUDENTS' INITIALS)			NOTES FOR NEXT STEPS
	STRONG	ADEQUATE	NEEDS WORK	
Concept Development • Records all data • Organizes data • Recognizes key features of frequency graph • Compares data • Changes categories and notes change in story				
Expressive Capacity • Uses precise language to describe the frequency graphs • Contributes to the discussion about the way changing categories affects the story the data tell				
Use of Tools • Uses graph paper • Uses atlas or other reference				
Background Knowledge • Draws on understanding of geography, relative size, and number sense for comparisons				

Rationale

The lesson takes students through the first steps of displaying data: collecting and organizing the information. As you work through this lesson, you and your students will notice that each time you categorize and recategorize the data set, you tell a different story.

Math Background

Data are information. This information may be numerical—e.g., salaries, test scores, heights, age, weight—as well as categorical—e.g., countries, foods we eat, types of product defects.

Most data are reported using statistics based on fractions and percents—one-third of pregnant mothers, half of native plants, etc. Commonly used, or benchmark, fractions and percents are often invoked to influence others to make decisions. When the headlines shout, "One-half of all pregnant women …," readers tend to form opinions based on the data. What readers often neglect to consider is the size of the sample, where the data were collected, and how the data were organized.

The media generally report data that are based on a sample. It is rare for any group to have the time or resources to ask every person in the population to contribute information. The idea of sampling is to study a part in order to gain information about the whole. The sample—its size and characteristics—influences the data and the conclusions of the study.

A representative sample by definition includes representation across the population being surveyed. For example, a representative sample of the community would include individuals from across town, not just one neighborhood. Likewise, a representative sample across America would include individuals from many different states and regions, not just one or two. The size of the sample is important when inferences are made. For example, 30 samples could be sufficient for a population of several hundreds. The deciding factor for determining sample size is how confident you want to be about the inference.

It is not the concern of this unit to determine appropriate sample size, but it is important to note that statisticians use formulas derived from repeated surveys to decide sample size. Regardless, small samples are used in many studies. Factors such as availability of subjects or scarcity of time or financial resources might lead researchers to use a small sample and to base policy or claims upon the outcome.

Throughout this unit, it will be important to help students begin to think critically about data—whether the data are grouped into categories of information or into graphs. They will need to understand that a random sample will often yield different results from one that is not random. This topic resurfaces in *Lesson 2*.

Context

Some students may know about *maquiladoras* in Mexican border towns, where women make clothes for very little money and with no benefits or environmental Occupational Safety and Health Administration (OSHA) workplace protections. CorpWatch (www.corpwatch.org) is one source for information on *maquiladoras*.

Facilitation

If students do not bring in data, or if their sample is too small, skip the second part of the *Opening Discussion*. Have available a pile of 20 clothing articles with labels. First, ask students to predict where the clothes were made. Post the list of their guesses. Note that it will be hard for them to answer this question unless they organize the information on the labels. Then divide up the 20 articles of clothing. Have students write the name of the country for each piece of clothing on a sticky note, one country name per note. Ask: "Where are most of our clothes made?" Then continue with the activity.

Making the Lesson Easier

Frequency graphs lend themselves to comparisons among categories. If students have little fluency in stating comparisons, you may choose to compare only size, using terms like "greater," "fewest," or "less than." For students who are encountering data formally for the first time, the notion that collapsing data yields different stories may be difficult. Treat this lightly in the activity, and revisit such questions after students have more experience categorizing and recategorizing data in the homework and in *Lesson 2*.

Making the Lesson Harder

If your students understand benchmark fractions and percents such as one half or 25%, get them to look critically at the data, including the source and sample size. You might ask:

If we asked another class what countries are in their closets, what do you think would happen to the categories? What if we asked the entire community?

How do you think your data would compare to data from another class of adult students in another community?

If students struggle with the idea of sample, you might try this: Have them each write their favorite color on a sticky note. If you have a small class, ask them to write the color on two sticky notes. Place all of the notes in a container. Have someone randomly (eyes closed) choose a few notes from the container and place them across a line to form a frequency graph. Ask the students how they think this sample compares to the actual total number of colors on notes in the container. Students can create another frequency graph using all of the notes and compare the sample to the actual total.

LESSON 1 IN ACTION

Alice articulates the mathematical principle behind compressed data.

I asked, "How did the change in categories affect what we noticed about the data?"

Alice answered, "Well, we keep losing information."

"How so?"

Patiently, Alice explained that when we started our work, every bit of data was visible. She added that we had lost details initially recorded. "At first, we knew every country in every person's closet and how many pieces of clothing came from that country. Then we combined the data, and we lost track of who had which countries. Then we did it by continent, and we lost track of all the countries."

Alice's realization quickly gained agreement from the rest of the class. After all, just the previous week a classmate had noted, "When you change the amount of data you look at, you find different things."

Sonia added her comment with increased conviction: "It is like politics. Politicians use a graph and tell you this is true, but you look at the graph, and it does not tell you everything."

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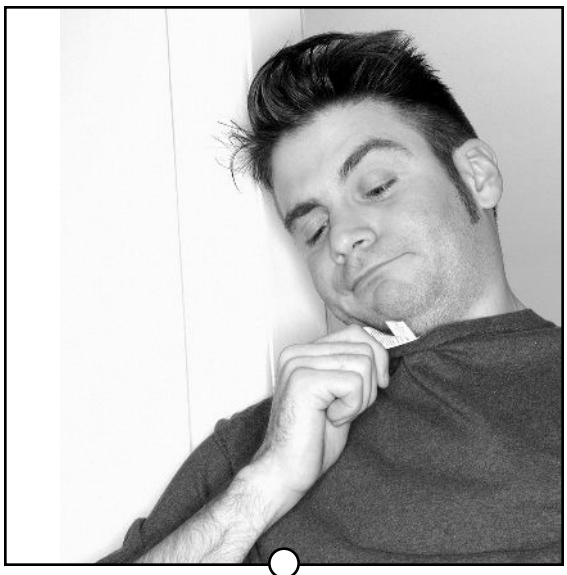
STUDENT BOOK

LESSON 1

Countries in Our Closets



Where were your clothes made?

In a world where information is at our fingertips, it is important to be able to make sense of it. What does it tell us? Why do we care?

Begin with your own clothes. Where were they made? Find out! You will collect **data** by reading the labels on your clothes. You will organize that data, make statements about it, and note where most of your clothes were made.

You will then try organizing the data in a different way. How will placing the data in new groups change the story of the countries in your closet? You will compare your class data to data of other classes. Will the stories be the same?

On Your Own

I think the country in which most of my clothes were made is

_____.

Look at the labels of at least eight items of clothing. Make a list to keep track of the information you are collecting. Research question: Where were your favorite clothes made?

Activity 1: Organizing the Data

Look at the data you collected about the countries where your clothes were made.

Organize the data.

1. What do you notice about your data?

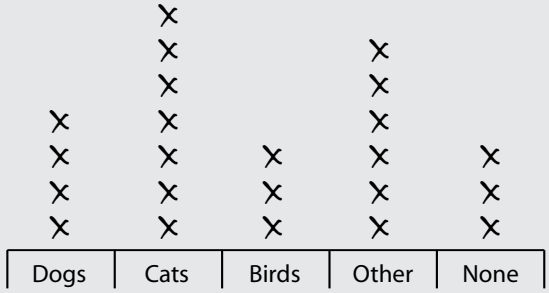
2. What is the **mode** for your data?

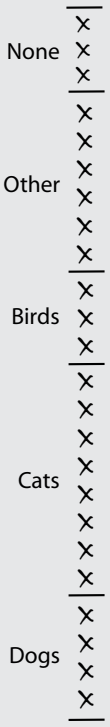
3. What is the mode for the class data?

Frequency Graphs

Create an easy-to-use **frequency graph** by making a line. Use equal-size x's and equal spacing for each **category**. Line up your x's or use graph paper so it is easy to note which category contains the most data items.

Some people can look at a frequency graph and make true statements by "eyeballing," or visually estimating, the size of each category. For example:





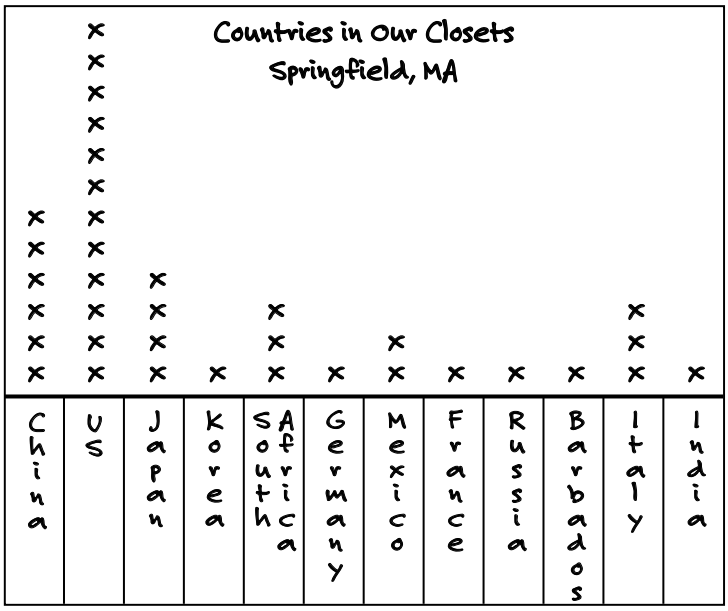
If you have a hard time visually estimating, try this: Count all the x's. Make one tall tower showing every x on the frequency graph. Compare each of the categories to the tower with all the data.

Does that category represent more than half, less than half, or exactly half of the data?

Compare two categories: Is one category more than half of the other?

Activity 2: Statements about Data

Use the data in the following frequency graph to fill in the blanks:



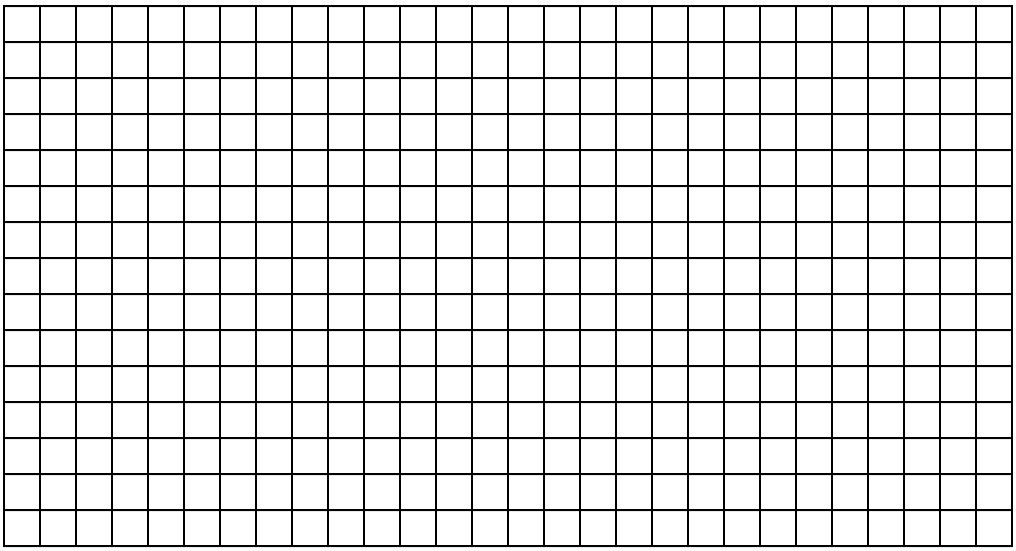
Use the following words to fill in the blanks:

half	six	United States	twice	three
one-fourth	four	one-third	Russia	China

1. There are _____ times as many clothes from South Africa as from Barbados.
2. The number of clothes from Mexico is _____ of the number of clothes from China.
3. _____ as many clothes come from the United States as from China.
4. The clothes from Korea are _____ as many as those from Japan.
5. _____ countries have only one article of clothing.
6. _____ of the clothes come from _____ and _____.
7. Japan has _____ times as many clothes as Germany.

Activity 3: Changing the Categories

On grid paper, make a new frequency graph using the class data. Group the clothes by continent this time, instead of by country.



Frequency graphs have three components: a line, frequencies (marked with x's), and labels. When making your own graph, be sure to include all three parts.

1. What do you notice about these new groupings (continents)?

2. How is the story of this graph different from the story of the whole-class frequency graph by country?

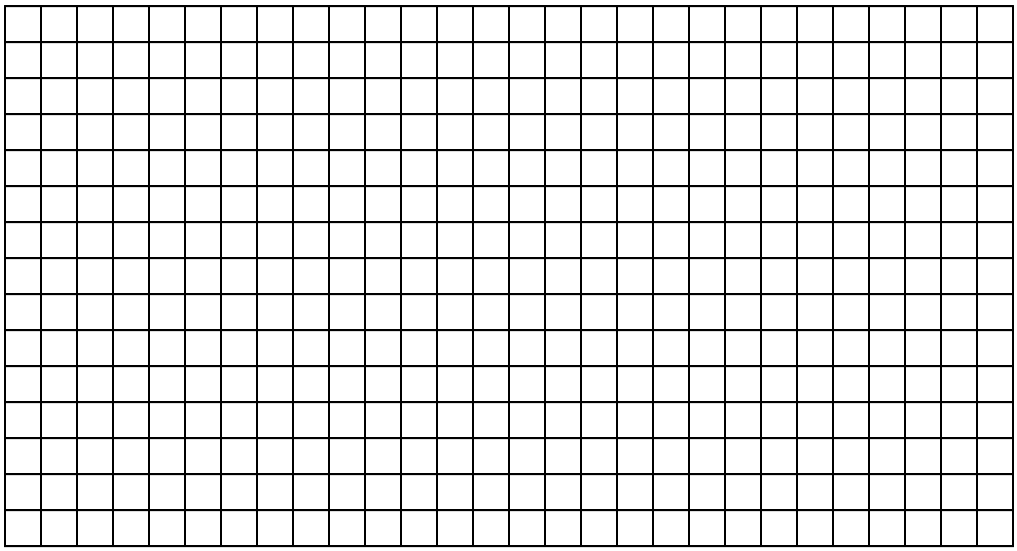
3. What advantages do you see to this way of grouping? What disadvantages?

4. Make a numerical statement about the new organization of the data.

Practice: Clothes by Continent in Springfield, MA

Refer to the Springfield data on page 10.

1. Sort the data by continent and make a frequency graph on the graph paper below:

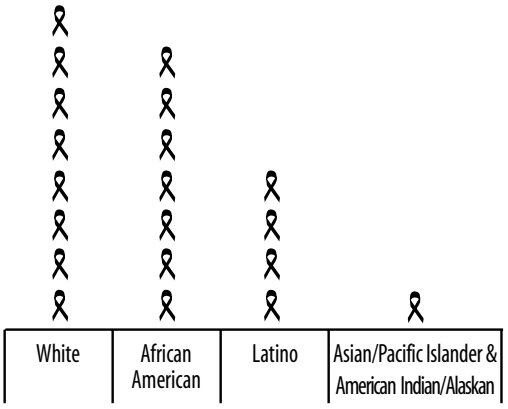


2. How do the data represented in the new graph compare to the data from the graph your class created?

3. Write one statement about the Springfield clothes frequency graph organized by continent.

Practice: Reporting Data 1

The race/ethnicity of 20 HIV/AIDS patients at a clinic in the United States is shown in the following frequency graph:



Source: Based on data from the AIDSHotline.org, 2003

1. Write a statement comparing two of the categories.

Ana reports the data from her clinic using only two categories: White and non-White.

2. Use grid paper to show what her frequency graph will look like.
3. How does using only two categories change the story of the HIV/AIDS data at her clinic?

4. Would you expect the data to be similar if the clinic were in your city, a city in Alaska, or a city in Florida?

5. Imagine a clinic in a city in your state. Show a frequency graph of 20 patients, categorized by race. Explain your thinking.

Practice: Reporting Data 2

The mayor wants to cut commuting time. He commissioned a survey to find out how long it takes people to get to work. The results were shown in five travel-time categories.

Regroup the data to show only three categories.

Travel Time to Work	Percent of Commuters	Number Based on 25 People
Less than 10 minutes	16%	4
10–19 minutes	32%	8
20–29 minutes	20%	5
30–44 minutes	20%	5
45 minutes or more	12%	3
Total	100%	25

Travel Time to Work	Percent of Commuters	Number Based on 25 People

Compare the two ways of organizing the information.

1. What is the travel-time category with the biggest percent of commuters?

   Five categories _____ Three categories _____

2. Which category has the smallest percent of people?

   Five categories _____ Three categories _____

3. How does regrouping the categories change your impression of people's travel time to work?

4. Which group would you recommend the mayor focus on if he starts a program to cut commuting time? Why?

Practice: Categorically Speaking

Sometimes we can organize categories into subgroups of other categories in order to better understand the data. For example, drug stores will often organize their aisles by category—skin care, beauty aids, and so on. Then the items in each of those aisles are also organized. This makes it easier to take inventory and place orders and for the customer to find products.

Can you think of other examples where information is categorized?

Use the chart below *or* create your own chart; draw a picture; make a list; or write about examples you see at home or at work. In the chart below, one example is given.

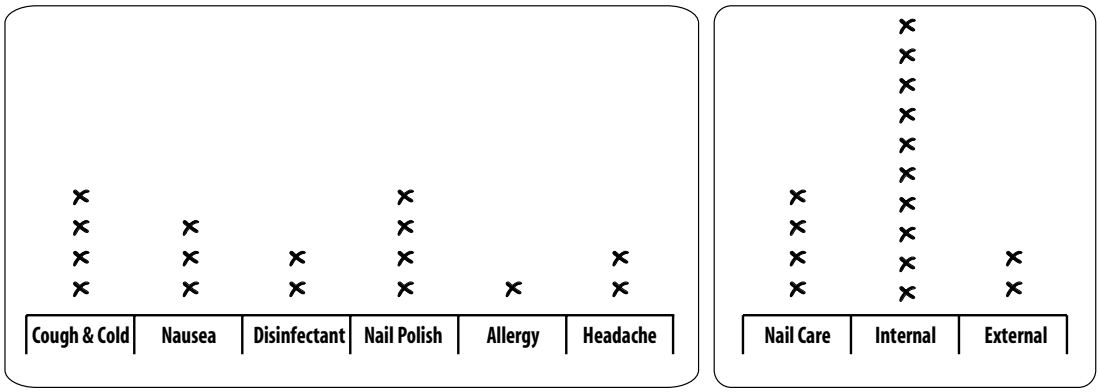
Location	**What Stuff or Information?**	**How Is It Organized?**
Drug store	Over-the-counter products they sell	Skin care Stationery Beauty aids First aid Seasonal Hair care

Extension: Taking Inventory

Find a drawer or closet filled with many things. Use a frequency graph to show the contents. Start with six categories. Then do another frequency graph to show the same items, but this time use three categories.

For example:

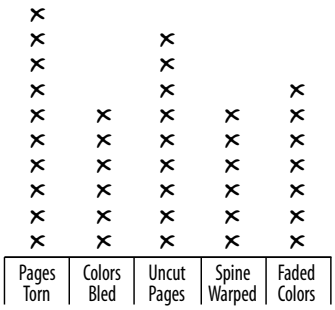
My Medicine Cabinet



Test Practice

1. Freda worked at a printing press. She kept a tally of all the defects she found in the books she was processing. Based on the frequency graph she created, shown below, which of the following statements is true?

Book Defects Frequency Graph



(1) There were more defects related to colors than any other defect.

(2) There were twice as many torn pages as there were warped spines.

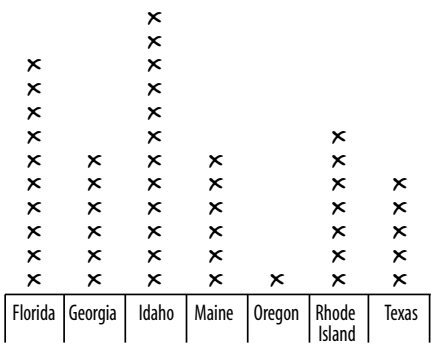
(3) There were more warped spines than colors that bled.

(4) Half of all the defects were related to colors that bled.

(5) Half of all the defects were related to pages that were torn.

2. A tour guide, has been keeping a tally of visitors from different states. According to his tally, which of the following is *not* a true statement?

U.S. Tourist Frequency Graph



(1) The fewest number of visitors came from Oregon.

(2) There were as many visitors from Maine as there were from Georgia.

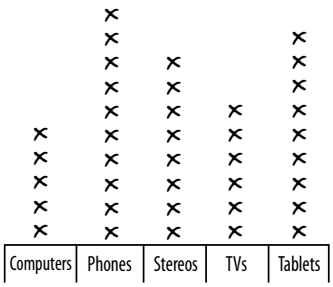
(3) There were as many visitors from Georgia as there were from Oregon and Texas.

(4) There were twice as many visitors from Florida as there were from Rhode Island.

(5) There were twice as many visitors from Idaho as there were from Maine.

3. Clara is paid a commission on each of the electronics items that she sells. She tallied the different items she sold for the month. Based on her tally, what can she tell her boss about her sales?

Electronics Sales Frequency Graph



(1) She sold twice as many phones as she did TVs.

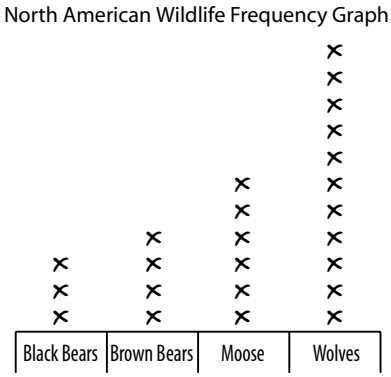
(2) She sold twice as many phones as she did computers.

(3) She sold more phones than she did TVs and computers combined.

(4) She sold more TVs and tablets than she did stereos and phones combined.

(5) She sold half as many TVs as she did stereos.

4. Bronson, a forest ranger in the Green Mountains, kept track of the different animals that were reportedly seen in one of the campgrounds during the month of May. Based on his tally, which of the following statements could he tell the media?



(1) There were twice as many wolves reportedly seen as bears.

(2) Half of all the reported sightings were wolves.

(3) There were more bears than wolves reportedly seen.

(4) There were twice as many moose reportedly seen than there were black bears.

(5) One-quarter of all reported sightings were brown bears.

5. According to one report, some workers in China who make clothing for a large U.S. company are paid as little as $0.16 per hour. At this wage, what amount would a worker make for a 40-hour workweek?

(1) $64.00

(2) $6.40

(3) $6.00

(4) $80.00

(5) $0.60

6. In 2003 the U.S. Department of Labor (DOL) issued new regulations for overtime pay. The DOL estimated that under the new regulations 1.3 million low-wage workers would become eligible for overtime pay, unless their wages were raised to $425 per week. The DOL estimated that 24.8 percent of those workers were Hispanic or Latino and 16.6 percent were Black or African American. What percent of workers were from neither of those two groups?